ARITHMETICAL SOLUTIONS

AND

MENSURATION

BY

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ARITHMETICAL

SOLUTIONS,

—AND—

MENSURATION:

COMPRISING

MODEL SOLUTIONS

WITH EXPLANATIONS.

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PREFACE.

WRITING makes the student exact. Many people have good ideas, but to be able to write them concisely one must have them clearly defined in his own mind, and then understand the rhetoric and the grammar of the language in which he writes.

It is much the same with mathematical solutions. Many teachers can get the answers to problems—they have the ideas, but they fail to have the solution of a problem standing out in their minds in all its logical proportions, each of the steps arranged according to its relation to the others. I have heard many teachers say, I can get the answer to that problem, and can understand it, but I can't arrange the solution so that it makes sense, of itself. The art of writing mathematical solutions has the same relation to mathematics that rhetoric has to literature.

We have endeavored to present a list of solutions in such a form as to enable those who study them to work, to a great extent, independent of rules and formulas.

It is a very common thing for teachers to show their pupils the rules and tell them to **work** by **them.** It is not uncommon to find teachers who, when asked "**why,**" will answer, "Because the rule says so." It will be seen at once that there is but little **discipline** or **originality** in following a rule. For example, the rule for finding what per cent. one number is of another, is as follows: **Divide the per centage by one per cent. of the base.** Very little dicipline is given to the mind by following this rule; the pupil knew before how to find one per cent, of a number; he also knew how to **divide.** Hence the method can claim but little merit in the way of educating the mind for original thought and investigation. Objections may be made to methods that would do away, to a great extent, with rules and "short-cuts," on the ground that such methods are not practical—that they do not meet the demands of business, etc. This may seem plausible, but two things are true:

1st. The pupil that knows a subject thoroughly and is master of its underlying principles can make short practical methods for himself.

2nd. The pupil who is taught the practical rules and short methods, is very likely not to know much of the underlying principles, and not master of the subject.

We have divided every solution in three general departments, indicated by the Roman chararacters—I., II., III. The first being the statement of the problem, the second its solution, and the third the conclusion. This is of little consequence, except so far as it may assist in leading the pupils to habits of classifying properly.

We call especial attention to Percentage and its applications. Those desiring to study these methods, would do well to follow their development in regular order, mastering each step as they go. Great care should be taken where one quantity is compared with another, whether in Percentage or not, to determine with **exactness** what is the **basis** of the **computation.** Percentage, in general, embraces all those computations involving per cent., and in a problem in which any number of per cent. is given, 100% is the **basis,** and the matter of **most importance, is to determine what that 100% represents.** In the discussion of problems under this subject, teachers should be rigid in requiring pupils to answer this question: "What does 100% represent in this problem?"

We are under lasting obligations to many pupils who have encouraged us, in the development of these methods, by approving them and carrying them into their schools.

December 20, 1886.

LEAST COMMON MULTIPLE and GREATEST COMMON DIVISOR.

I. Find the L. C. M. and G. C. D. of 20, 25, 30.

II.
1. $20 = 2 \times 2 \times 5$.
2. $25 = 5 \times 5$.
3. $30 = 2 \times 3 \times 5$.
4. L. C. M. $= 2 \times 2 \times 5 \times 5 \times 3 = 300$.
5. G. C. D. $= 5$

III. ∴ L. C. M. = 300. G. C. D. = 5.

EXPLANATION: 2 must be used as a factor of Least Common Multiple **twice**, else the number found would not contain 20. 5 must be used twice, else the number found would not contain 25. 3 must be used **once**, else the number would not contain 30. Hence the Least Common Multiple $=2\times2\times5\times5\times3$. By examining this set of factors it will be seen that the respective factors of each of the numbers are to be found in it.

GREATEST COMMON DIVISOR FOUND BY DIVISION.

Find Greatest Common Divisor of 150 and 35.

SOLUTION:

```
35 ) 150 ( 4
     140
     ---
      10 ) 35 ( 3
           30
           --
            5 ) 10
                --
                 2
```

EXPLANATION: We desire to find a number that will divide 35 and 150. It is evident that any number that will divide 35 will divide each of the 35's that are in 150. By dividing 150 by 35 we find that there are four 35's in 150; these may be discarded, since any number that will divide 35 will divide each of the 35's that are in 150. Hence we need only use 35 and the 10 that **remains** after the 35's are discarded. The same reasoning applies to 10 and 35 as did to 35 and 150. We divide by 10 to discard the 10's that are in 35, knowing that any number that will divide 10 will divide all the 10's in 35's and so on.

I. Find three numbers that have 1050 for their L. C. M. and 10 for their G. C. D.

$$\text{II.}\begin{cases} 1.\quad 1050 = 5 \times 5 \times 2 \times 3 \times 7. \\ 2.\quad 10 = 2 \times 5. \\ 3.\quad 2 \times 5 \times 5 = 50 = \text{1st}. \\ 4.\quad 2 \times 5 \times 3 = 30 = \text{2nd}. \\ 5.\quad 2 \times 5 \times 7 = 70 = \text{3rd}. \end{cases}$$

III. ∴ 50, 30, and 70 = required numbers.

EXPLANATION: 10 is the Greatest Common Divisor, hence its factors 2 and 5 are found in **each of the numbers.** This gives us **two factors** in each number. By examining the factors of Least Common Multiple it will be seen that 5 is used **twice** in at least **one** of the required numbers.

Again, we see that another of the required numbers has, besides 2 and 5, the factor 3; hence use 2, 5 and 3 as its factors. The third number has, besides the factor 1 and 5, the factor 7; hence use 2, 5 and 7 as its factors.

It may be asked why not use as factors of the first, 2, 5, 5 and 3? Because we would then have **one** factor, 7, left to assist in getting two more numbers—which would not be sufficient. Were two numbers required, that would do.

GENERAL OBSERVATIONS: (1.) 2 and 5 are factors of each of the numbers.

(2.) 5 is found twice in **at least one** of the numbers, but it **may** be found twice in **all but one.**

(3.) 2 is found once in **each** number; 3 and 7 may be found in all but one.

(4.) From this it will be seen that other numbers may be found that will satisfy the conditions of the problem.

I. What two numbers between 35 and 840 have the former for their G. C. D. and the latter for their L. C. M?

$$\text{II.}\begin{cases} 1.\quad 840 = 2 \times 2 \times 2 \times 3 \times 5 \times 7. \\ 2.\quad 35 = 5 \times 7. \\ 3.\quad 5 \times 7 \times 2 \times 2 \times 2 = 280 = \text{first number}. \\ 4.\quad 5 \times 7 \times 3 = 105 = \text{second number}. \end{cases}$$

III. ∴ 280 and 105 are the required numbers.

R. H. p. 96.

NOTE: 5 and 7 must be used as factors of each number, because they are the factors of the Greatest Common Divisor. 2 being found 3 times in the Least Common Multiple indicates that it is used 3 times in one of the numbers.

FRACTIONS.

I. REDUCTIONS.

I. Reduce $8\frac{3}{4}$ to an improper fraction.

II. 1. $8\frac{3}{4} = 8 + \frac{3}{4}$.
2. $1 = \frac{4}{4}$.
3. $8 = 8$ times $\frac{4}{4} = \frac{32}{4}$.
4. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$.

III. $\therefore 8\frac{3}{4} = \frac{35}{4}$.

I. Reduce $\frac{4}{5}$ to thirtieths.

II. 1. $\frac{5}{5} = \frac{30}{30}$.
2. $\frac{1}{5} = \frac{1}{5}$ of $\frac{30}{30} = \frac{6}{30}$.
3. $\frac{4}{5} = 4$ times $\frac{6}{30} = \frac{24}{30}$.

III. $\therefore \frac{4}{5} = \frac{24}{30}$.

I. Reduce $\frac{2}{7}$ to twelfths.

II. 1. $\frac{7}{7} = \frac{12}{12}$.
2. $\frac{1}{7} = \frac{1}{7}$ of $\frac{12}{12} = \frac{\frac{12}{7}}{12}$.
3. $\frac{2}{7} = 2$ times $\frac{\frac{12}{7}}{12} = \frac{\frac{24}{7}}{12} = \frac{3\frac{3}{7}}{12}$.

III. $\therefore \frac{2}{7} = \frac{3\frac{3}{7}}{12}$.

I. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ to L. C. Denominator.

II. 1. L. C. D. $= 12$.
2. $\frac{1}{2} = \frac{6}{12}$.
3. $\frac{2}{3} = \frac{8}{12}$.
4. $\frac{3}{4} = \frac{9}{12}$.

III. $\therefore \frac{1}{2}, \frac{2}{3}, \frac{3}{4} = \frac{6}{12}, \frac{8}{12}, \frac{9}{12}$.

FUNDAMENTAL OPERATIONS,

I. ADDITION.

I. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

II.
1. $12 =$ L. C. Denominator.
2. $\frac{2}{3} = \frac{8}{12}$.
3. $\frac{3}{4} = \frac{9}{12}$.
4. $\frac{5}{6} = \frac{10}{12}$.
5. $\frac{8}{12} + \frac{9}{12} + \frac{10}{12} = \frac{27}{12} = 2\frac{1}{4}$.

III. $\therefore \frac{2}{3} + \frac{3}{4} + \frac{5}{6} = 2\frac{1}{4}$.

II. SUBTRACTION.

I. From $\frac{5}{6}$ take $\frac{3}{10}$.

II.
1. L. C. Denominator $= 30$.
2. $\frac{5}{6} = \frac{25}{30}$.
3. $\frac{3}{10} = \frac{9}{30}$.
4. $\frac{25}{30} - \frac{9}{30} = \frac{16}{30} = \frac{8}{15}$.

III. $\therefore \frac{5}{6} - \frac{3}{10} = \frac{8}{15}$.

III. MULTIPLICATION.

I. $\frac{8}{13} \times \frac{11}{16} =$ what?

II.
1. $\frac{8}{13} \times \frac{16}{16} = \frac{8}{13}$.
2. $\frac{8}{13} \times \frac{8}{16} = \frac{1}{16}$ of $\frac{8}{13} = \frac{1}{26}$.
3. $\frac{8}{13} \times \frac{11}{16} = 11$ times $\frac{1}{26} = \frac{11}{26}$.

III. $\therefore \frac{8}{13} \times \frac{11}{16} = \frac{11}{26}$.

NOTE:—This is on the principle, that as the multiplier decreases the product decreases, and as the multiplier increases the product increases.

IV. DIVISION.

I. $\frac{3}{4} \div \frac{4}{5} =$ what?

II.
1. $\frac{3}{4} \div \frac{5}{5} = \frac{3}{4}$.
2. $\frac{3}{4} \div \frac{1}{5} = 5$ times $\frac{3}{4} = \frac{15}{4}$.
3. $\frac{3}{4} \div \frac{4}{5} = \frac{1}{4}$ of $\frac{15}{4} = \frac{15}{16}$.

III. $\therefore \frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$.

NOTE:—This is on the principle, that the less the divisor the greater the quotient, the greater the divisor the less the quotient.

MISCELLANEOUS PROBLEMS.

I. What number divided by $\frac{3}{5}$ will give 10 for a quotient?

II.
1. $\frac{5}{5}$ = required number.
2. $\frac{5}{5} \div \frac{3}{5} = \frac{5}{3}$ = quotient.
3. 10 = quotient. Hence
4. $\frac{5}{3} = 10$.
5. $\frac{1}{3} = \frac{1}{5}$ of $10 = 2$.
6. $\frac{3}{3} = 3$ times $2 = 6$.

III. ∴ 6 = the number required.

NOTE: In the equation (2) $\frac{5}{3}$ = quotient. In equation (3) 10 = quotient. Two things that equal the same thing, equal each other. This axiom is of great service in solution of problems.

I. What number is that, to which if you add $\frac{3}{7}$ of itself the sum will be 20?

II.
1. $\frac{7}{7}$ = required number.
2. $\frac{7}{7} + \frac{3}{7} = \frac{10}{7}$.
3. $\frac{10}{7} = 20$.
4. $\frac{1}{7} = \frac{1}{10}$ of $20 = 2$.
5. $\frac{7}{7} = 7$ times $2 = 14$.

III. ∴ 14 = required number.

R. 3rd p. 164.

I. A has \$2,400; $\frac{5}{8}$ of his money + \$500 is $\frac{5}{4}$ of B's; what sum has B?

II.
1. \$2,400 = A's money.
2. $\frac{5}{8}$ of \$2,400 = \$1,500.
3. \$1,500 + \$500 = \$2,000.
4. $\frac{4}{4}$ = B's money.
5. $\frac{5}{4}$ = \$2,000.
6. $\frac{1}{4} = \frac{1}{5}$ of \$2,000 = \$400.
7. $\frac{4}{4} = 4$ times \$400 = \$1,600.

III. ∴ B has \$1,600.

R. 3rd p. 164.

I. $\frac{6}{7}$ is $\frac{3}{8}$ of what number?

II.
1. $\frac{8}{8}$ = the number.
2. $\frac{3}{8} = \frac{6}{7}$.
3. $\frac{1}{8} = \frac{1}{3}$ of $\frac{6}{7} = \frac{2}{7}$.
4. $\frac{8}{8} = 8$ times $\frac{2}{7} = \frac{16}{7} = 2\frac{2}{7}$.

III. ∴ $\frac{6}{7}$ is $\frac{3}{8}$ of $2\frac{2}{7}$.

I. A merchant owning $\frac{4}{15}$ of a vessel sold $\frac{1}{2}$ of his share for \$1,640; what was the vessel worth at that rate?

II.
1. $\frac{1}{2}$ of $\frac{4}{15} = \frac{2}{15}$ = part sold.
2. \$1,640 = value of part sold.
3. $\frac{2}{15} = \$1,640$.
4. $\frac{1}{15} = \frac{1}{2}$ of $\$1,640 = \$820$.
5. $\frac{15}{15} = 15$ times $\$820 = \$12,300$.

III. ∴ The vessel is worth \$12,300.

It will be observed that he did not sell ½ of the vessel for \$1,640, but ½ of his share.

I. A and B together have \$620; $\frac{3}{4}$ of A's money equals $\frac{4}{5}$ of B's; how much has each?

II.
1. $\frac{3}{4}$ A's $= \frac{4}{5}$ B's.
2. $\frac{1}{4}$ A's $= \frac{1}{3}$ of $\frac{4}{5}$ B's $= \frac{4}{15}$ B's.
3. $\frac{4}{4}$ A's $= 4$ times $\frac{4}{15}$ B's $= \frac{16}{15}$ B's.
4. $\frac{15}{15}$ = B's money.*
5. $\frac{16}{15}$ = A's money.
6. $\frac{15}{15} + \frac{16}{15} = \frac{31}{15}$ = what both have.
7. \$620 = what both have.
8. $\frac{31}{15} = \$620$.
9. $\frac{1}{15} = \frac{1}{31}$ of $\$620 = \$20$.
10. $\frac{15}{15} = 15$ times $\$20 = \$300$.
11. $\frac{16}{15} = 16$ times $\$20 = \$320$.

III. ∴ A's = \$320.
B's = \$300.

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*Equations (1), (2) and (3) simply find the value of A's in terms of B's. The 3rd could be read as follows: A's money equals $\frac{16}{15}$ of B's. Where two quantities are compared and one is given in terms of the other,

the one that follows the preposition OF **is the** BASIS **of the calculation.** Hence in this problem B's money is **the basis**, and is $\frac{15}{15}$. The same explanation may be applied to the solution immediately following. In all solutions, where quantities are **compared** in this way, it is of great importance to be able to tell with **certainty** what is the basis of **calculation.** We could reason on the (3) and (4) as follows: The 3rd states that A's is $\frac{16}{15}$ of B's, then B's is $\frac{15}{15}$ of his own.

1. The sum of A's and B's ages is 41 years; $\frac{2}{5}$ of A's age equals $\frac{5}{8}$ of B's; find the age of each.

II.
1. $\frac{2}{5}$ A's $= \frac{5}{8}$ B's.
2. $\frac{1}{5}$ A's $= \frac{1}{2}$ of $\frac{5}{8}$ B's $= \frac{5}{16}$ B's.
3. $\frac{5}{5}$ A's $=$ 5 times $\frac{5}{16}$ B's $= \frac{25}{16}$ B's.
4. $\frac{16}{16} =$ B's age.
5. $\frac{25}{16} =$ A's age.
6. $\frac{16}{16} + \frac{25}{16} = \frac{41}{16} =$ sum.
7. 41 years $=$ sum. Hence,
8. $\frac{41}{16} =$ 41 years.
9. $\frac{1}{16} = \frac{1}{41}$ of 41 years $=$ 1 year.
10. $\frac{16}{16} =$ 16 times 1 year $=$ 16 years $=$ B's age.
11. $\frac{25}{16} =$ 25 times 1 year $=$ 25 years $=$ A's age.

III. ∴ A's age $=$ 25 years.
B's age $=$ 16 years.

I. The sum of two numbers is 120; one equals 3 times the other; find the numbers.

II.
1. $\frac{3}{3} =$ lesser.
2. $\frac{9}{3} =$ greater.
3. $\frac{9}{3} + \frac{3}{3} = \frac{12}{3} =$ both.
4. $\frac{12}{3} =$ 120.
5. $\frac{1}{3} = \frac{1}{12}$ of 120 $=$ 10.
6. $\frac{3}{3} =$ 3 times 10 $=$ 30.
7. $\frac{9}{3} =$ 9 times 10 $=$ 90.

III. ∴ 30 $=$ one.
90 $=$ other.

I. The sum of the ages of A and B is 53 years; $3\frac{1}{2}$ times A's age equals $5\frac{1}{3}$ times B's age; find age of each.

II.
1. $3\frac{1}{2}=\frac{7}{2}$; $5\frac{1}{3}=\frac{16}{3}$.
2. $\frac{7}{2}$ A's $=\frac{16}{3}$ B's.
3. $\frac{1}{2}$ A's $=\frac{1}{7}$ of $\frac{16}{3}$ B's $=\frac{16}{21}$ B's.
4. $\frac{2}{2}$ A's $=2$ times $\frac{16}{21}$ B's $=\frac{32}{21}$ B's.
5. $\frac{21}{21}=$ B's age.
6. $\frac{32}{21}=$ A's age.
7. $\frac{21}{21}+\frac{32}{21}=\frac{53}{21}=$ sum of ages.
8. 53 years $=$ sum of ages.
9. $\frac{53}{21}=53$ years.
10. $\frac{1}{21}=\frac{1}{53}$ of 53 years $=1$ year.
11. $\frac{21}{21}=21$ times 1 year $=21$ years $=$ B's age.
12. $\frac{32}{21}=32$ times 1 year $=32$ years $=$ A's age.

III. ∴ A's age $=32$ years.
B's age $=21$ years.

I. A person being asked the time of day replied that $\frac{2}{5}$ of the time past noon equaled $\frac{3}{4}$ of the time to midnight; find the time of day.

It will be observed that this problem does **not give** either the time past noon, or time to midnight in **terms** of the other, but gives a part of one equal to a part of the other. Hence it cannot be told which is the **basis** until we reduce them so as to have one **in terms** of the other.

II.
1. $\frac{2}{5}$ t. p. n. $=\frac{3}{4}$ t. t. m.
2. $\frac{1}{5}$ t. p. n. $=\frac{1}{2}$ of $\frac{3}{4}$ " $=\frac{3}{8}$ t. t. m.
3. $\frac{5}{5}$ t. p. n. $=5$ times $\frac{3}{8}$ " $=\frac{15}{8}$ of t. t. m.
4. $\frac{8}{8}=$ time to midnight.
5. $\frac{15}{8}=$ time past noon.
6. $\frac{8}{8}+\frac{15}{8}=\frac{23}{8}=$ time from noon to midnight.
7. 12. hours $=$ time from noon to midnight.
8. $\frac{23}{8}=12$ hours.
9. $\frac{1}{8}=\frac{1}{23}$ of 12 hours $=\frac{12}{23}$ hours.
10. $\frac{15}{8}=15$ times $\frac{12}{23}$ hrs. $=7\frac{19}{23}$ hrs. $=$ t. p. n.

III. ∴ $7\frac{19}{23}$ o'clock P. M. $=$ time of day.

NOTE. The time of day between noon and midnight is always stated as the number of hours **after noon**, and not the number of hours to midnight; hence in the (10) we use $\frac{15}{8}$ instead of $\frac{8}{8}$, since by the (5) we see that $\frac{15}{8}$ represents the number of hours **past noon.**

I. A person being asked the time of day said that $\frac{2}{3}$ of the time past noon equaled the time to midnight; find the time of day.

II.
1. $\frac{3}{3}=$ time past noon.
2. $\frac{2}{3}=$ time to midnight.
3. $\frac{3}{3}+\frac{2}{3}=\frac{5}{3}$ time from noon to midnight.
4. 12 hours $=$ time from noon to midnight.
5. $\frac{5}{3}=12$ hours.
6. $\frac{1}{3}=\frac{1}{5}$ of 12 hours $=\frac{12}{5}$ hours.
7. $\frac{3}{3}=3$ times $\frac{12}{5}=\frac{36}{5}$ hours $=7\frac{1}{5}$ hours.

III. ∴ It is $7\frac{1}{5}$ hours past noon, $7\frac{1}{5}$ o'clock, P. M.

In this problem the time to midnight is given in **terms** of the time past noon. That is, the time to midnight is ⅔ **of** the time past noon. Notice what follows the preposition **of**.

A person giving the time of day said, that $\frac{3}{4}$ of the time to midnight equaled the time past noon; fine the time of day.

In this example the time to midnight follows the preposition of; hence the **basis** is the time to midnight, and would be represented by $\frac{4}{4}$, while ¾ would be the time past noon, and their sum would be the time from noon to midnight, which is 12 hours.

I. The sum of A's and B's money is \$1,600; A has $\frac{2}{3}$ more than B; how much has each?

II.
1. $\frac{3}{3}=$ B's money.
2. $\frac{3}{3}+\frac{2}{3}=\frac{5}{3}=$ A's money.
3. $\frac{3}{3}+\frac{5}{3}=\frac{8}{3}=$ what both have.
4. \$1,600 $=$ what both have.
5. $\frac{8}{3}=\$1{,}600$.
6. $\frac{1}{3}=\frac{1}{8}$ of $\$1{,}600=\$200$.
7. $\frac{3}{3}=3$ times $\$200=\$600=$ B's.
8. $\frac{5}{3}=5$ times $\$200=\$1{,}000=$ A's.

III. ∴ A's $=\$1{,}000$; B's $=\$600$.

This represents a class of problems in which one quantity is a certain part **more than** or **less than** the other. In such examples the quantity following the expression "more than" or "less than," is the basis. The latter part of the above problem means that A has $\frac{2}{3}$ of B's more than B's.

I. A man spent $\frac{2}{3}$ of $\frac{4}{5}$ more than half his money, and had $140 left; how much had he at first?

II.
1. $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.
2. $\frac{2}{2} =$ his money.
3. $\frac{1}{2} =$ half his money.
4. $\frac{8}{15}$ of $\frac{1}{2} = \frac{4}{15}$.
5. $\frac{1}{2} + \frac{4}{15} = \frac{23}{30} =$ part spent.
6. $\frac{30}{30} - \frac{23}{30} = \frac{7}{30} =$ part left.
7. $\frac{7}{30} = \$140$.
8. $\frac{1}{30} = \frac{1}{7}$ of $\$140 = \$20$.
9. $\frac{30}{30} = 30$ times $\$20 = \$600$.

III. ∴ He had $600.

REMARK: $\frac{8}{15}$ more than $\frac{1}{2}$ is $\frac{8}{15}$ of $\frac{1}{2}$, plus $\frac{1}{2}$.

I. A man spent $\frac{1}{5}$ more than $\frac{2}{3}$ of his money and had $300 left; how much did he have?

II.
1. $\frac{3}{3} =$ his money.
2. $\frac{1}{5}$ of $\frac{2}{3} = \frac{2}{15}$.
3. $\frac{2}{3} + \frac{2}{15} = \frac{12}{15} = \frac{4}{5} =$ amount spent.
4. $\frac{5}{5} - \frac{4}{5} = \frac{1}{5} =$ amount left.
5. $\frac{1}{5} = \$300$.
6. $\frac{5}{5} = 5$ times $\$300 = \$1,500$.

III. ∴ $1,500 = his money.

REMARK: In this example it is stated that he spent $\frac{1}{5}$ more than $\frac{2}{3}$ of his money; $\frac{1}{5}$ more than $\frac{2}{3}$ is $\frac{1}{5}$ of $\frac{2}{3}$ plus $\frac{2}{3}$ which is $\frac{12}{15}$ his money.

I. A man has two watches, and a chain worth \$20; if he puts the chain on the first watch, it will be worth $\frac{2}{3}$ as much as the second watch; but if he puts the chain on the second watch, it will be worth $2\frac{3}{4}$ times the first watch; find value of each watch.

II.
1. $\frac{2}{3}$ s $=$ $\frac{3}{3}$ f $+$ \$20.
2. $\frac{1}{3}$ s $\frac{1}{2}$ $=$ of ($\frac{3}{3}$ f $+$ \$20) $=$ $\frac{1}{2}$ f $+$ \$10.
3. $\frac{3}{3}$ s $=$ 3 times ($\frac{1}{2}$ f $+$ \$10) $=$ $\frac{2}{3}$ f $+$ \$30.
4. $\frac{2}{2}$ $=$ 1st watch.
5. $\frac{3}{2}$ f $+$ \$30 $=$ 2nd watch.
6. $\frac{11}{4}$ f $=$ $\frac{3}{2}$ f $+$ \$50.
7. $\frac{1}{4}$ f $=$ $\frac{1}{11}$ of ($\frac{3}{2}$ f $+$ \$50) $\frac{3}{22}$ f $+$ $\$\frac{50}{11}$.
8. $\frac{4}{4}$ f $=$ 4 times ($\frac{3}{22}$ f $+$ $\$\frac{50}{11}$) $=$ $\frac{6}{11}$ f $+$ $\$\frac{200}{11}$.

(8.) $-$ $\frac{6}{11}$ f $=$ (9.) $\frac{5}{11}$ f $=$ $\$\frac{200}{11}$.

10. $\frac{1}{11}$ f $=$ $\frac{1}{5}$ of $\$\frac{200}{11}$ $=$ $\$\frac{40}{11}$.
11. $\frac{11}{11}$ f $=$ 11 times $\$\frac{40}{11}$ $=$ \$40 $=$ 1st.

(1.) $=$ (12.) $\frac{2}{3}$ s $=$ \$40 $+$ \$20 $=$ \$60.

13. $\frac{1}{3}$ s $=$ $\frac{1}{2}$ of \$60 $=$ \$30.
14. $\frac{3}{3}$ s $=$ 3 times \$30 $=$ \$90 $=$ 2nd.

III. $\therefore$ Cost of 1st $=$ \$40.
Cost of 2nd $=$ \$90.

W. p. 243.

In the eighth equation we see $\frac{4}{4}$ or $\frac{11}{11}$ of the first watch on one side of the equation and $\frac{6}{11}$ of first watch on the other side. Of course $\frac{6}{11}$ of first watch can be subtracted from both sides, which results in the ninth equation. The 11th equation gives the value of first watch. This placed instead of $\frac{3}{3}$ f in the 1st equations, gives the 12th equation.

I. If $\frac{5}{8}$ of 24 be 12, what will $\frac{6}{7}$ of 21 be?

II.
1. $\frac{5}{8}$ of 24 $=$ 15.
2. $\frac{6}{7}$ of 21 $=$ 18. By the supposition
3. 15 $=$ 12.
4. 1 $=$ $\frac{1}{15}$ of 12 $=$ $\frac{4}{5}$.
5. 18 $=$ 18 times $\frac{4}{5}$ $=$ $\frac{72}{5}$ $=$ $14\frac{2}{5}$.

III. $\therefore$ Under the supposition, $\frac{6}{7}$ of 21 $=$ $14\frac{2}{5}$.

I. Ten years ago the age of A was $\frac{3}{4}$ of the age of B, and ten years hence the age of A will be $\frac{5}{6}$ of the age of B; find the age of each.

II.
1. $\frac{4}{4}$ = B's age 10 years ago.
2. $\frac{3}{4}$ = A's age 10 years ago.
3. $\frac{4}{4}$ + 20 years = B's age 10 years hence.
4. $\frac{3}{4}$ + 20 years = A's age 10 years hence.
5. $\frac{5}{6}$ of ($\frac{4}{4}$ + 20 years) = $\frac{5}{6}$ B's + $16\frac{2}{3}$ years.
6. $\frac{5}{6}$ B's + $16\frac{2}{3}$ years = $\frac{3}{4}$ B's + 20 years.

(6.)—$\frac{3}{4}$ B's = (7.) $\frac{1}{12}$ B's + $16\frac{2}{3}$ yrs. = 20 yrs.

(7.)—$16\frac{2}{3}$ years = (8.) $\frac{1}{12}$ B's = $3\frac{1}{3}$ years.

9. $\frac{12}{12}$ = 12 times $3\frac{1}{3}$ = 40 yrs. = B's age 10 yrs. ago.
10. $\frac{3}{4}$ of 40 yrs. = 30 yrs. = A's age 10 yrs ago.
11. 40 yrs. + 10 yrs. = 50 yrs = B's age now.
12. 30 yrs. + 10 yrs. = 40 yrs. = A's age now.

III. ∴ 50 years = B's age.
40 years = A's age.

In the 5th equation we see $\frac{5}{6}$ B's age on one side, and $\frac{3}{4}$ B's age on the other. We can subtract $\frac{3}{4}$ B's age from both sides and affect the equation only in form. We also subtract $16\frac{2}{3}$ years from both sides of the 6th equation. By making these subtractions we simplify the equations.

I. At the time of marriage a wife's age was $\frac{3}{5}$ of that of her husband, and 10 years after marriage her age was $\frac{7}{10}$ the age of her husband; how old was each at the time of marriage?

II.
1. $\frac{5}{5}$ = husband's age at time of marriage.
2. $\frac{3}{5}$ = wife's age at time of marriage.
3. $\frac{5}{5}$ + 10 yrs. = age of hus. 10 yrs. after mar.
4. $\frac{3}{5}$ + 10 yrs. = age of wife 10 yrs. after mar.
5. $\frac{7}{10}$ of ($\frac{5}{5}$ h. + 10 years) = $\frac{7}{10}$ h. + 7 years.
6. $\frac{7}{10}$ h. + 7 years = $\frac{3}{5}$ h. + 10 years.

(6.)—$\frac{3}{5}$ h. = (7.) $\frac{1}{10}$ h. + 7 years = 10 years.

(7.)—7 years = (8.) $\frac{1}{10}$ h. = 3 years.

9. $\frac{10}{10}$ h. = 10 times 3 years = 30 years = his age.
10. $\frac{3}{5}$ of 30 years = 18 years = wife's age.

III. ∴ 30 years = husband's age at marriage.
18 years = wife's age at marriage.

EXAMPLES.

1. Two men, A and B, bought a farm paying \$6,000 for it; A paid $\frac{3}{4}$ as much as B; what sum did each pay?

2. $\frac{3}{4}$ of a number increased by $\frac{2}{11}$ less than $\frac{2}{9}$ of it, is 400; find the number.

3. A person giving the time of day said that $\frac{2}{5}$ of time to midnight equaled $\frac{5}{9}$ of the time past noon; find the time of day.

4. Being asked the time of day, I said that $\frac{3}{8}$ time past noon equaled 4 times the time to midnight; find the time of day.

5. A man spent $\frac{2}{7}$ of $\frac{9}{10}$ less than $\frac{3}{4}$ his money and had \$600 left; how much had he at first?

6. $\frac{3}{4}$ of my money equals $\frac{2}{5}$ of it plus \$210; how much have I?

7. $\frac{7}{8}$ of B's age plus 10 years equals $\frac{4}{7}$ of his age plus 27 years; how old is he? See remark page 14.

8. A boy lost $\frac{4}{5}$ of his money and then found 80 cents; he then had 20 cents less than what he had in the beginning; how much had he at first?

9. 15 years ago A's age was $\frac{3}{5}$ of B's; 20 years, hence A's will be $\frac{5}{6}$ of B's; find age of each.

10. $\frac{3}{5}$ of a pole is in the mud; $\frac{1}{4}$ of it in the water and 10 feet in the air; how many feet are in the mud?

METRIC SYSTEM.

Let the teacher present this subject in the following order:

1st. Begin with the **four measuring units**, METER, ARE, LITER and GRAM, and get it thoroughly fixed in the pupil's mind that—

Meter measures length.
Are measures surface.
Liter measures capacity.
Gram measures weight.

2nd. Show that ARE, LITER and GRAM are derived from the METER.

3rd. Drill pupils **rigidly** on the following prefixes:

$$\text{Latin, }\left\{\begin{array}{rl}\text{milli} &= .001.\\ \text{centi} &= .01.\\ \text{deci} &= .1.\end{array}\right. \qquad \text{Greek, }\left\{\begin{array}{rl}\text{deka} &= 10.\\ \text{hecto} &= 100.\\ \text{kilo} &= 1000.\\ \text{myria} &= 10000.\end{array}\right.$$

4th. Now apply these prefixes to the measuring units as follows:

Milli-meter = one thousandth of a meter.
Centi-liter = one hundredth of a liter.
Deka-liter = 10 liters, and so on.

By the above the teacher can lead the pupils to make all the necessary tables for themselves. Let us make a table of weight, using the GRAM as the unit of measure. Beginning with the Latin prefixes it will be seen that a milli is only $\frac{1}{10}$ of a centi, or a milli-gram is only $\frac{1}{10}$ of a centi-gram; hence 10 milli-gram = 1 centi-gram. Again, a centi is $\frac{1}{10}$ of a deci, or a centi-gram is $\frac{1}{10}$ of a deci-gram; hence 10 centi-grams = 1 deci-gram, and so on to the Greek prefixes. In these we see that deka is $\frac{1}{10}$ of hecto, or a deka-gram = $\frac{1}{10}$ of a hecto-gram; hence 10 deka-grams = 1 hecto-gram, and so on.

COMPOUND NUMBERS.

The object in presenting this subject, is to show that the processes used in adding, subtracting, multiplying and dividing compound numbers are exactly the same as those used in addition, subtraction, multiplication and division of simple numbers.

Add the following:

		4	8
Compound Numbers.	4 bushels,	3 pecks,	7 quarts.
	5 "	2 "	5 "
	6 "	3 "	4 "
	2 "	3 "	3 "
	2 "	2 "	2 "
	22 bushels,	3 pecks,	5 quarts.

		10	10
Simple Numbers.	4 hundreds,	3 tens,	7 units.
	5 "	2 "	5 "
	6 "	3 "	4 "
	2 "	3 "	3 "
	2 "	2 "	2 "
	20 hundreds,	5 tens,	1 unit=2051

By examining the last example it will be seen that in adding simple numbers, we add each column, and divide the sum by the number of units of that order, required to make one of the next higher, place the remainder under the column, and carry the quotient to the next higher order. Precisely the same is done with common numbers. Adding the right-hand column of the second we have $7+5+4+3+2=21$; $21 \div 10=2$, with a remainder 1. In the first example, by adding the right-hand column we have just the same sum, 21; this we divide by 8, because 8 of that order are required to make one of the next higher. The only difference then is in the relation of the orders. In simple numbers that relation is **always** 10, while in compound numbers it varies. The same explanation applies to subtraction.

Multiply the following compound number by 4.

7 24

Compound No.—5 weeks, 9 days, 8 hours.
4

25 weeks, 2 days, 8 hours.

Multiply the following simple number by 4.

Simple No.—5 hundreds, 9 tens, 8 units.
4

23 hundreds, 9 tens, 2 units = 2392.

NOTE. The process in these is exactly the same. In the compound number we multiply 8 hours by 4 and divide by 24 to reduce to days; in the simple number we multiply the 8 units by 4 and divide by 10 to reduce to tens. In the compound number we multiply 9 days by 4, add the one day derived from the hours, and divide by 7 to reduce to weeks; in the simple number we multiply 9 tens by 4, add the tens derived from the units and divide by 10 to reduce to hundreds, and so on. Thus we see the **processes** are the same. The same is true of Division.

PERCENTAGE.

PERCENTAGE PROPER.

In Percentage **three** quantities are considered: (1) **Base,** (2) the **Rate,** and (3) the **Percentage.** When **two** of these quantities are given, the third may be found.

Analytical solutions will be given, of examples under the **four** following cases.

CASE I.

$$\text{Given}\left\{\begin{array}{l}\text{Base,}\\ \text{Rate,}\end{array}\right\}\text{required Percentage.}$$

$$\text{I. Find } 20\% \text{ of } \$500.$$

$$\text{II.}\left\{\begin{array}{ll}1. & 100\% = \$500.\\ 2. & 1\% = \tfrac{1}{100} \text{ of } \$500 = \$5.\\ 3. & 20\% = 20 \text{ times } \$5 = \$100.\end{array}\right.$$

$$\text{III.} \therefore 20\% \text{ of } \$500 = \$100.$$

$$\text{I. Find } \tfrac{5}{8}\% \text{ of } \$600.$$ X

$$\text{II.}\left\{\begin{array}{ll}1. & 100\% = \$600.\\ 2. & 1\% = \tfrac{1}{100} \text{ of } \$600 = \$6.\\ 3. & \tfrac{5}{8}\% = \tfrac{5}{8} \text{ of } \$6 = \$3.75.\end{array}\right.$$

$$\text{III.} \therefore \tfrac{5}{8}\% \text{ of } \$600 = \$3.75.$$

$$\text{I. Find } .2\% \text{ of } .3\% \text{ of } .3 \text{ of } \$1{,}500.$$

$$\text{II.}\left\{\begin{array}{l}\text{A}\left\{\begin{array}{ll}1. & .3 \text{ of } \$1{,}500 = \$450\\ 2. & 100\% = \$450.\\ 3. & 1\% = \tfrac{1}{100} \text{ of } \$450 = \$4.50.\\ 4. & .3\% = .3 \text{ of } \$4.50 = \$1.35.\end{array}\right.\\ \text{B}\left\{\begin{array}{ll}1. & 100\% = \$1.35.\\ 2. & 1\% = \tfrac{1}{100} \text{ of } \$1.35 = \$.0135.\\ 3. & .2\% = .2 \text{ of } \$.0135 = \$.0027.\end{array}\right.\end{array}\right.$$

④

$$\text{III.} \therefore .2\% \text{ of } .3\% \text{ of } .3 \text{ of } \$1{,}500 = \$.0027.$$

NOTE: This solution, to be logically arranged, must be divided into two departments, A and B; for it will be observed that 100% is used twice, and represents a **different sum** each time; at one time it equals \$450, and at another \$1.35.

CASE II.

Given $\left\{ \begin{matrix} \text{Base,} \\ \text{Percentage,} \end{matrix} \right\}$ required Rate.

REMARK: The Base may be known by its being the quantity with which the other is compared, and will follow the expression "of," "more than," or "less than." See top of page 12.

I. What per cent. of 8 is 2?

II. 1. $8 = 100\%$.
2. $1 = \frac{1}{8}$ of $100\% = 12\frac{1}{2}\%$.
3. $2 = 2$ times $12\frac{1}{2}\% = 25\%$.

III. ∴ 2 is 25% of 8.

NOTE: In this example 8 is the basis and is 100% of itself.

I. What per cent. of 3 is 9?

II. 1. $3 = 100\%$.
2. $1 = \frac{1}{3}$ of $100\% = 33\frac{1}{3}\%$.
3. $9 = 9$ times $33\frac{1}{3} = 300\%$.

III. ∴ 9 is 300% of 3.

I. What per cent. of $\frac{3}{8}$ is $\frac{2}{15}$? 0 √

II. 1. $\frac{3}{8} = 100\%$.
2. $\frac{1}{8} = \frac{1}{2}$ of $100\% = 50\%$.
3. $\frac{3}{8} = 3$ times $50\% = 150\%$.
4. $\frac{2}{15} = \frac{2}{15}$ of $150\% = 20\%$.

III. ∴ $\frac{2}{15}$ is 20% of $\frac{3}{8}$.

CASE III.

Given $\left\{ \begin{matrix} \text{Rate,} \\ \text{Percentage,} \end{matrix} \right\}$ required Base.

I. 60 is 20% of what number?

II. 1. $100\% =$ number required.
2. $20\% = 60$.
3. $1\% = \frac{1}{20}$ of $60 = 3$.
4. $100\% = 100$ times $3 = 300$.

III. ∴ 60 is 20% of 300.

I. $\frac{3}{4}$ is 250% of what number?

II.
1. 100% = required number.
2. 250% = $\frac{3}{4}$.
3. 1% = $\frac{1}{250}$ of $\frac{3}{4}=\frac{3}{1000}$.
4. 100% = 100 times $\frac{3}{1000}=\frac{3}{10}$.

III. ∴ $\frac{3}{4}$ is 250% of $\frac{3}{10}$.

R. 3rd p. 202.

I. $14\frac{2}{7}$ is $16\frac{2}{3}\%$ of what number?

II.
1. $14\frac{2}{7}=\frac{100}{7}$; $16\frac{2}{3}\%=\frac{50}{3}\%$.
2. 100% = required number.
3. $\frac{50}{3}\%=\frac{100}{7}$.
4. $\frac{1}{3}\%=\frac{1}{50}$ of $\frac{100}{7}=\frac{2}{7}$.
5. $\frac{3}{3}\%=3$ times $\frac{2}{7}=\frac{6}{7}$.
6. 100% = 100 times $\frac{6}{7}=\frac{600}{7}=85\frac{5}{7}$.

III. ∴ $14\frac{2}{7}$ is $16\frac{2}{3}\%$ of $85\frac{5}{7}$.

I. A man owning 60% of a factory, sold 40% of his share for \$9,600; at this rate, what was the value of the factory?

II.
1. 100% = value of factory.
2. 60% = his share.
3. 40% of 60% = 24% = part sold.
4. \$9,600 = value of part sold.
5. 24% = \$9,600.
6. 1% = $\frac{1}{24}$ of \$9,600 = \$400.
7. 100% = 100 times \$400 = \$40,000 = val. of fac.

III. ∴ \$40,000 = value of factory.

Wh. Com. p. 141.

I. The number of pupils in daily attendance in a certain school is 420, which is 80 per cent. of the enrollment; how many pupils are enrolled?

II.
1. 100% = number enrolled.
2. 80% = number in daily attendance.
3. 420 = number in daily attendance.
4. 80% = 420.
5. 1% = $\frac{1}{80}$ of 420 = 5.25.
6. 100% = 100 times 5.25.

III. ∴ 525 = number enrolled.

Wh. Com. p. 141.

CASE IV.

Given { Rate and sum or difference of Base and Percentage, } required Base.

I. What number diminished by 6 per cent. of itself is equal to 7.52?

II.
1. 100% = number.
2. 100% — 6% = 94%.
3. 94% = 7.52.
4. 1% = $\frac{1}{94}$ of 7.52 = .08.
5. 100% = 100 times .08 = 8.

III. ∴ 8 = the required number.

R. 3rd, p. 203.

I. A horse cost $160, which was 20% less than the cost of the carriage; what was the cost of carriage?

See Remark, page 20.

II.
1. 100% = cost of carriage.
2. 100% — 20% = 80% = cost of horse.
3. $160 = cost of horse.
4. 80% = $160.
5. 1% = $\frac{1}{80}$ of $160 = $2.
6. 100% = 100 times $2 = $200.

III. ∴ Carriage cost $200.

Wh. Com. p. 142.

I. A school enrolls 230 boys, which is 15% more than the number of girls enrolled; how many pupils in the school?

See Remark, page 20.

II.
1. 100% = number of girls.
2. 100% + 15% = 115% = number of boys.
3. 230 = number of boys.
4. 115% = 230.
5. 1% = $\frac{1}{115}$ of 230 = 2.
6. 100% = 100 times 2 = 200 = number of girls.
7. 230 + 200 = 430 = number of pupils.

III. ∴ 430 = number of pupils enrolled.

Wh. Com. p. 142.

MISCELLANEOUS EXAMPLES.

REMARKS. It has already been remarked that when one thing is said to be such a part "of," or so much "more than," or "less than," another, the one following such expression is the base. But if it is said that a part of one is equal to a part of another, then it will be necessary to find the one in terms of the other, and then the base can be determined. The following for example: 20% of A's money is 25% of B's. Reduce as follows:

$$\begin{aligned} &20\%\text{ A's} = 25\%\text{ B's.}\\ &1\%\text{ A's} = \tfrac{1}{20}\text{ of }25\%\text{ B's} = \tfrac{5}{4}\%\text{ B's.}\\ &100\%\text{ A's} = 100\text{ times }\tfrac{5}{4}\%\text{ B's} = 125\%. \end{aligned}$$

Now this last reads that A's is 125% of B's, hence B's is the base and is 100%.

I. A and B together have \$500; 10% of A's equals 15% of B's; how much has each?

$$\text{II.}\left\{\begin{aligned} &1.\quad 10\%\text{ A's} = 15\%\text{ B's.}\\ &2.\quad 1\%\text{ A's} = \tfrac{1}{10}\text{ of }15\%\text{ B's} = \tfrac{3}{2}\%\text{ B's.}\\ &3.\quad 100\%\text{ A's} = 100\text{ times }\tfrac{3}{2}\%\text{ B's} = 150\%\text{ B's.}\\ &4.\quad 100\% = \text{B's.}\\ &5.\quad 150\% = \text{A's.}\\ &6.\quad 250\% = \text{sum of both.}\\ &7.\quad \$500 = \text{sum of both.}\\ &8.\quad 250\% = \$500.\\ &9.\quad 1\% = \tfrac{1}{250}\text{ of }\$500 = \$2.\\ &10.\quad 100\% = 100\text{ times }\$2 = \$200 = \text{B's.}\\ &11.\quad 150\% = 150\text{ times }\$2 = \$300 = \text{A's.} \end{aligned}\right.$$

$$\text{III.}\ \therefore\ \left\{\begin{aligned} &\text{A's} = \$300.\\ &\text{B's} = \$200. \end{aligned}\right.$$

NOTE: The 3rd equation reads that A's equals 150% of B's; B's share is 100% of itself, and A's is 150% of B's share.

I. A miller takes for toll 6 quarts for every 5 bushels of wheat ground; what per cent. does he take?

II.
1. 5 bushels = 160 quarts.
2. 160 quarts = 100%.
3. 1 quart = $\frac{1}{160}$ of 100% = $\frac{5}{8}$%.
4. 6 quarts = 6 times $\frac{5}{8}$% = $3\frac{3}{4}$%.

III. ∴ He takes $3\frac{3}{4}$% toll.

R. 3rd, p. 203.

NOTE: Had the miller taken 160 quarts, he would have taken all, or 100%. Hence 160 = 100%.

I. A farmer owning 45% of a tract of land, sold 540 acres, which was 60% of what he owned; how many acres were there in the tract?

II.
1. 100% = tract of land.
2. 45% = what he owned.
3. 60% of 45% = 27% = part sold.
4. 540 acres = part sold.
5. 27% = 540 acres.
6. 1% = $\frac{1}{27}$ of 540 acres = 20 acres.
7. 100% = 100 times 20 acres = 2,000 a. = tract.

III. ∴ The tract = 2,000 acres.

R. 3rd, p. 204.

I. The five-cent piece weighs 5 grams, of which 1.25 grams are nickel, and the remainder copper; what per cent. of it is copper?

II.
1. 5 grams = both, (nickel and copper).
2. 1.25 grams = nickel.
3. 5 grams—1.25 grams = 3.75 grams = copper.
4. 5 grams = 100%.
5. 1 gram = $\frac{1}{5}$ of 100% = 20%.
6. 3.75 grams = 3.75 times 20% = 75%.

III. ∴ 75% is copper.

R. 3rd, p. 205.

I. A man's expenses are 80% of his income, and $33\frac{1}{3}\%$ of his income equals 10% of his property, which is valued at $27,000; what are his expenses?

II.
1. $27,000 = his property.
2. 10% of $27,000=$2,700=$33\frac{1}{3}\%$ of income.
3. 100% = income.
4. $33\frac{1}{3}\% = \$2,700$.
5. $1\% = \frac{1}{33\frac{1}{3}}$ of $2,700 = $81.
6. 100% = 100 times $81 = $8,100.
7. 80% of $8,100 = $6,480 = expenses.

III. ∴ His expenses are $6,480.

Wh. Com. p. 145.

NOTE: In the 3rd equation we might let $\frac{3}{3}$ = income and in the 4th $\frac{1}{3}$ = $2,700. Then $\frac{3}{3}$ =3 times $2,700 = $8.100.

I. If 32% of 75% of 800% of a number is 1,539, what is it?

II.
1. 100% = the required number.
2. 75% of 800% = 600%.
3. 32% of 600% = 192%.
4. 192% = 1,539.
5. $1\% = \frac{1}{192}$ of 1,539 = $8.01\frac{9}{16}$.
6. 100% = 100 times $8.01\frac{9}{16} = 801\frac{9}{16}$.

III. ∴ $801\frac{9}{16}$ = required number.

R. H., p. 210.

I. A farmer raises wheat and corn; his wheat crop is worth $1,036, which is 40% more than the value of the corn crop; what is the value of the corn crop?

II.
1. 100% = value of corn crop.
2. 100% + 40% = 140%.
3. 140% = $1,036.
4. $1\% = \frac{1}{140}$ of $1,036 = $7.60.
5. 100% = 100 times $7.60 = $760.

III. ∴ The corn was worth $760.

I. A coat cost \$32; the trimmings cost 75% less and the making 50% less than the cloth; what did each cost?

II.
1. 100% = value of cloth.
2. 100% = 75% = 25% = cost of trimmings.
3. 100% − 50% = 50% = cost of making.
4. 100% + 25% + 50% = 175% = cost of coat.
5. \$32 = cost of coat.
6. 175% = \$32.
7. 1% = $\frac{1}{175}$ of \$32 = \$.1828*x*
8. 100% = 100 times \$.1828 = \$18.28*x* = cloth.
9. 25% = \$4.57*x* = trimmings.
10. 50% = \$9.14*x* = making.

III. ∴ \$17.77*x* = cost of cloth.
\$ 5.33*x* = cost of trimmings.
\$ 8.84*x* = cost of making.

R. H., p. 211.

I. An army loses 27 per cent. of its number in battle and has 22,630 men remaining; how many did it contain?

II.
1. 100% = required number.
2. 27% = number lost.
3. 100% − 27% = 73% = number remaining.
4. 22,630 = number remaining.
5. 73% = 22,630.
6. 1% = $\frac{1}{73}$ of 22,630 = 310.
7. 100% = 100 times 310 = 31,000.

III. ∴ 31,000 = number of men in the army.

I. 81 men are 5% of 60% of how many men?

II.
1. 100% = number of men.
2. 5% of 60% = 3%.
3. 3% = 81 men.
4. 1% = $\frac{1}{3}$ of 81 men = 27 men.
5. 100% = 100 times 27 men = 2,700 men.

III. ∴ 81 men = 5% of 60% of 2,700 men.

APPLICATIONS OF PERCENTAGE.

COMMISSION.

'21

I. A wool agent receives a commission of 2%, and his commission is $40; find the value of the wool handled.

II. 1. 100% = value of wool.
2. 2% = commission.
3. $40 = commission.
4. 2% = $40.
5. 1% = $\frac{1}{2}$ of $40 = $20.
6. 100% = 100 times $20 = $2,000.

III. ∴ The value of the wool was $2,000.

a.

I. An agent receives $210 with which to purchase goods; after deducting his commission at 5% what sum will he spend for goods?

II. 1. 100% = amount spent for goods.
2. 5% = commission.
3. 105% = entire amount received.
4. $210 = entire amount received.
5. 105% = $210.
6. 1% = $\frac{1}{105}$ of $210 = $2.
7. 100% = 100 times $2 = $200.

III. ∴ $200 = amount spent for goods.

I. What are the net proceeds on a sale of goods amounting to $180, at 4% commission?

II. 1. 100% = entire amount of sale.
2. 4% = commission.
3. 100% − 4% = 96% = net proceeds.
4. 96% of $180 = $172.80.

III. ∴ $172.80 = net proceeds.

3

I. A lawyer collected a bill at 25% commission and remitted \$7.50 as net proceeds; what was the amount collected?

II.
1. 100% = amount collected.
2. 25% = commission.
3. 100% — 25% = 75% = net proceeds.
4. \$7.50 = net proceeds.
5. 75% = \$7.50.
6. 1% = $\frac{1}{75}$ of \$7.50 = \$.10.
7. 100% = 100 times \$.10 = \$10.

III. ∴ \$10 = the amount collected.

4

I. A society paid a lad \$6 for collecting membership dues, to the amount of \$100; what rate of commission did he receive?

II.
1. \$100 = 100%.
2. \$1 = $\frac{1}{100}$ of 100% = 1%.
3. \$6 = 6 times 1% = 6%.

III. ∴ He received 6% commission.

I. A real estate broker sold a section of land (640 acres) at \$7.50 per acre, and invested the proceeds in railroad stock, receiving $1\frac{1}{2}$% for selling the land and $\frac{7}{8}$% for buying the stock; what was his brokerage?

II.

A
1. 640 acres at \$7.50 = \$4,800.
2. $1\frac{1}{2}$% of \$4,800 = \$72 = com. for selling.
3. \$4,800—\$72 = \$4,728 = net proceeds.

B
1. 100% = am'nt of railroad stock bought.
2. $\frac{7}{8}$% = commission.
3. $100\frac{7}{8}$% = railroad stock + commission
4. \$4,728 = railroad stock + commission.
5. $100\frac{7}{8}$% = \$4,728.
6. 1% = $\frac{6}{100\frac{7}{8}}$ of \$4,728 = \$46.869*x*.
7. $\frac{7}{8}$% = $\frac{7}{8}$ of \$46.869*x* = \$41.01*x* = com.

III. ∴ His commission was \$72 + \$41.01 = \$113.01.

I. A grain dealer in Chicago received $5,000 with directions to purchase wheat at $1.10 a bushel, after deducting his commission at 2½%; how many bushels of wheat did he purchase?

II.
1. 100% = amount spent for wheat.
2. 2½% = commission.
3. 102½% = amount received.
4. $5,000 = amount received.
5. 102½% = $5,000.
6. 1% = 1/102½ of $5,000 = $48.78.
7. 100% = 100 times $48.78 = $4,878.
8. $4,878÷$1.10=4434.5=number of bushels.

III. ∴ He purchased 4434.5 bushels.

I. My commission at 3% was $120; find amount of sale.

II.
1. 100% = amount of sale.
2. 3% = commission.
3. $120 = commission.
4. 3% = $120.
5. 1% = ⅓ of $120 = $40.
6. 100% = 100 times $40 = $4,000.

III ∴ $4,000 = amount of sale.

TRADE DISCOUNT.

I. Bought 100 dozen stay bindings at 60 cents per dozen, for 40, 10 and 7½% off; what did I pay for them?

II.
1. 1 dozen = 60 cents.
2. 100 dozen = 100 times 60 cents = $60.
3. 40% of $60 = $24.
4. $60 — $24 = $36.
5. 10% of $36 = $3.60.
6. $36 — $3.60 = $32.40.
7. 7½% of $32.40 = $2.43.
8. $32.40—$2.43 = $29.97.

III ∴ $29.97 = cost.

I. Sold 20 dozen feather dusters, giving the purchaser a discount of 10, 10 and 10%; his discounts amounting to $325.20; how much was my price per dozen?

II.
1. 100% = my price for 20 dozen.
2. 10% of 100% = 10% = 1st discount.
3. 100% — 10% = 90%.
4. 10% of 90% = 9% = 2nd discount.
5. 90% — 9% = 81%.
6. 10% of 81% = 8.1% = 3rd discount.
7. 10%+9%+8.1%=27.1%=am't of disc'ts.
8. $325.20 = amount of discounts.
9. 27.1% = $325.20.
10. 1% = $\frac{1}{27.1}$ of $325.20 = $12.
11. 100% = 100 times $12 = $1,200.
12. 20 dozen = $1,200.
13. 1 dozen = $\frac{1}{20}$ of $1,200 = $60.

III. ∴ $60 = price per dozen.

R. 3rd, p. 290.

PROFIT AND LOSS.

I. Sold silk at $1.35 per yard, and lost 10 per cent.; at what price per yard would I have sold to make profit of 16⅔ per cent.?

II.

A
1. 100% = cost price.
2. 10% = loss.
3. 90% = selling price.
4. $1.35 = selling price.
5. 90% = $1.35.
6. 1% = $\frac{1}{9}$ of $1.35 = $.015.
7. 100% = 100 times $.015 = $1.50 = cost.

B
1. 100% = $1.50.
2. 1% = $\frac{1}{100}$ of $1.50 = $.015.
3. 16⅔% = 16⅔ times $.015 = $.25.
4. $1.50 + $.25 = $1.75.

III. ∴ $1.75 = price sold at to gain 16⅔%.

R. 3rd, p. 211.

I. A bookseller sells a grammar for 90 cents which cost 75 cents; what is his gain per cent.?

II.
1. 75 cents = cost price.
2. 90 cents = selling price.
3. 90 cents — 75 cents = 15 cents = gain.
4. 75 cents = 100%.
5. 1 cent = $\frac{1}{75}$ of 100% = $\frac{4}{3}$%.
6. 15 cents = 15 times $\frac{4}{3}$% = 20%.

III. ∴ He gained 20%.

R. 3rd, p. 210.

I. A merchant reduced the price of a certain piece of cloth 5 cents per yard, and thereby reduced his profit on the cloth from 10% to 8%; what was the cost of the cloth per yard?

II.
1. 100% = cost.
2. 10% = profit before deducting.
3. 8% = profit after deducting.
4. 10% — 8% = 2% = deduction.
5. 5 cents = deduction.
6. 2% = 5 cents.
7. 1% = $\frac{1}{2}$ of 5 cents = $\frac{5}{2}$ cents.
8. 100% = 100 times $\frac{5}{2}$ cents = $2.50.

III. ∴ $2.50 = cost price.

R. 3rd, p. 211.

I. A merchant sells goods at retail 30% above cost, and at wholesale 12% less than the retail price; what is his gain per cent. on goods sold at wholesale?

II.
1. 100% = cost.
2. 130% = retail.
3. 12% of 130% = 15.6%.
4. 130% — 15.6% = 114.4% = wholesale price.
5. 114.4%—100%=14.4%=gain at wholesale.

III. ∴ He gained 14.4% at wholesale.

Wh. p. 148.

I. A merchant sold velvet at a profit of $2 per yard and gained 20 per cent.; how much did it cost?

II.
1. 100% = cost per yard.
2. 20% = gain.
3. $2 = gain.
4. 20% = $2.
5. 1% = $\frac{1}{20}$ of $2 = $.10.
6. 100% = 100 times $.10 = $10.

III. ∴ $10 = price of velvet.

I. How must cloth, costing $3.50 a yard, be marked that a merchant may deduct 15 per cent. from the marked price and still make 15 per cent. profit?

II.
A
1. $3.50 = cost price.
2. 15% = gain.
3. 15% of $3.50 = $.525.
4. $3.50 + $.525 = $4.025 = selling price.

B
1. 100% = marked price.
2. 15% = deduction.
3. 100% — 15% = 85% = selling price.
4. 85% = $4.025.
5. 1% = $\frac{1}{85}$ of $4.025 = $.04735.
6. 100% = 100 times $.04735 = $4.735.

III. ∴ $4.735 = marked price.

Wh. p. 148.

REMARK: In the 1st department of this solution 100% = cost; in the second, 100% = marked price; hence the solution is divided into two departments.

I. A merchant marked a piece of silk at 25 per cent. above cost, and then sold it at 20 per cent. less than marked price; did he gain or lose, and how much?

II.
1. 100% = cost.
2. 125% = marked price.
3. 20% of 125% = 25%.
4. 125% — 25% = 100% = selling price.
5. 100% — 100% = 0.

III. ∴ He neither gained nor lost.

I. How must I mark goods costing $7 per yard that I may deduct 40 per cent. from the asking price and still make 20 per cent.?

II.
A
1. $\$7 =$ cost.
2. $20\% =$ gain.
3. 20% of $\$7 = \$1.40 =$ gain.
4. $\$7 + \$1.40 = \$8.40 =$ selling price.

B
1. $100\% =$ asking price.
2. $40\% =$ reduction.
3. $100\% - 40\% = 60\% =$ selling price.
4. $60\% = \$8.40$.
5. $1\% = \frac{1}{60}$ of $\$8.40 = \$.14$.
6. $100\% = 100$ times $\$.14 = \$14$.

III. ∴ I must mark it at $14.

I. A merchant bought tea at 20 per cent. less than its market value, and received a discount of 5 per cent. for cash. He sold it at an advance of 15 per cent. above its market value; what was his gain per cent.?

II.
A
1. $100\% =$ market value.
2. $80\% =$ price to merchant.
3. 5% of $80\% = 4\%$ discount.
4. $80\% - 4\% = 76\% =$ actual cost to merch.

B
1. $115\% =$ selling price.
2. $76\% =$ actual cost.
3. $115\% - 76\% = 39\%$.
4. $76\% = 100\%$ of itself.
5. $1\% = \frac{1}{76}$ of $100\% = \frac{100}{76}\% = \frac{25}{19}\%$.
6. $39\% = 39$ times $\frac{25}{19}\% = 51\frac{6}{19}\%$.

III. ∴ His gain was $51\frac{6}{19}$ per cent.

REMARK: Under A we found 76% of the market value to be the actual cost to the merchant. It is stated that he sold it at 115% of the market value; hence he gained 39% of the market value, but his actual gain on his investment is whatever per cent. 39% is of 76% which is found by the 4th, 5th and 6th equations under B.

I. Sold an article at 20 per cent. gain; had it cost me $300 more, I would have lost 20 per cent.; find the cost.

II.
1. 100% = actual cost.
2. 100% + 20% = 120% = actual selling price.
3. 100% + $300 = supposed cost.
4. 20% = loss on supposed cost.
5. 20% of (100% + $300) = 20% + $60 = loss.
6. (100% + $300) - (20% + $60) = 80% + $240 = s.p
7. 120% = selling price.
8. 120% = 80% + $240.
(8.)—80% = (9.) 40% = $240.
10. 1% = $\frac{1}{40}$ of $240 = $6.
11. 100% = 100 times $6 = $600.

III. ∴ $600 = cost price.

R. H., p. 369.

REMARK: In the 5th equation we find that 20%+$60 = loss on the supposed cost. If we deduct this from the supposed cost the result will be the selling price, which was 120% of the actual cost. Hence the 8th equation. Taking 80% from both sides of the 8th we have the 9th. Below we give a solution of this example which was given to us by a teacher. We publish it to illustrate bad methods.

100 + 20% = $120. $300 — $120 = $180.
180 ÷ 1% = 20 &c. = 20)180($900.
180
——
$900—$300=$600=Ans.

COMMISSION AND BROKERAGE COMBINED.

(1.) A commission man may get commission for **buying, selling** or **collecting.**

The important thing to keep constantly in mind is the **base,** or the thing upon which commission is computed. For instance, if he is buying he gets commission on what he **spends.** 2nd. If **selling,** then his commission is on **selling price;** if **collecting,** the commission is on the **sum collected.**

(2.) A **broker** is one who buys or sells whatever has a **par** value. The broker is usually defined as one who buys or sells stocks, bonds, **etc.** This "etc." is not definite enough. The teacher can get the idea of **par** value before his class by questions. For example ask the class to name some things that have **par** value. The pupils will say notes, stocks, money, and so on. Then ask them to name somethings that have **no** par value. That is easily done. Now at this place get clearly before them the difference between brokerage and commission—the one has to do with what has a **par value,** while the other has to do with that which **has no par value.**

Here are some examples such as may be used by the teacher to to enforce the main principles of these subjects:

(1.) My agent bought silk for me 10% below list, and charged me 2% for buying; what did the silk cost ME?

1. 100% = list.
2. 90% = what agent paid.
3. 2% of 90% = 1.8% = agents commission.
4. 90%+1.8%=91.8%=cost to me in terms of par.

REMARK: He, the agent, is buying, and his commission is on what he pays.

(2.) My agent buys silk 20% below list, and charges commission of 5%. I sell the goods 10% below list, what per cent. profit do I make on my investment?

1. 100% = list.
2. 80% = what agent paid.
3. 5% of 80% = 4% = commission.

∴ 4. 84% = cost to me.

5. 90% = selling price (sold at 10% below list).

∴ 6. 14% = gain.

Now whatever per cent. 14 is of 84 will be the per cent. gain; 14 per cent. is not the per cent. gain, because it is in terms of **par.** The per cent. gain or loss is always in terms of cost. 14% is 16⅔% of 84%.

(3.) My agent bought silk 20% below list, commission 3%, and I sold it through another agent at 10% below list, his commission being 3%, what per cent. profit did I make on my investments?

1. 100% = par.
2. 80% = what agent paid.
3. 3% of 80% = 2.4% = commission.
4. 82.4% = total cost to me.
5. 90% = what other agent sold it at.
6. 3% of 90% = 2.7% = commission.
7. 87.3% = net sale.
8. 87.3% — 82.4% = 4.9% = gain.

Now find what per cent. 4.9 is of 82.4 and this will be the per cent. gain.

REMARK: At this point when your pupils think they understand this principle of commission well, let the teacher slip in an example like the following:

(4.) Got my broker to buy me some bonds, which he secured at 10% below par, charging me 2% for buying, and then at my advice he sold them 5% above par, charging me 2% for selling; what per cent. profit did I make on my investment?

Your class will quite likely solve it wrong, as follows:

1. 100% = par.
2. 90% = what broker paid.
3. 2% of 90% = 1.8% = brokerage.
4. 91.8% = total cost.
5. 105% = what broker got when he sold bonds.
6. 2% of 105% = 2.1% = brokerage.
7. 102.9% = net sale.
8. 102.9% — 91.8% = 11.1% = gain, and then find what per cent. 11.1% is 91.8%.

You tell them this is wrong, and let them study brokerage and find out why.

The point is that while in commission the agent gets his per cent. commission on the business done, yet in brokerage the broker gets his per cent. on **par value always**, and accordingly the following is the **correct solution** for the above example:

1. 100% = par.
2. 90% = what broker paid.
3. 2% ≐ brokerage.
4. 92% = total cost.
5. 105% = what broker sold for.
6. 103% = net sale.
7. 103% — 92% = 11% = gain.

And whatever per cent. 11 is of 92 is the per cent. gain.

REMARK: Nothing can have a better effect than for the teacher constantly to impress upon the minds of his pupils that per cent. gain or loss is always estimated on **cost**. That commission is always on **cost** if agent is buying, on selling price if agent is selling, and on amount collected if agent is collecting; that brokerage is **always** on par value. These things are fundamental and until they are firmly fixed in the mind the learner knows but little of these subjects.

I. A broker buys 17 shares Milwaukee and St. Paul preferred stock; what is his brokerage at $\frac{1}{4}\%$?

II. 1. 1 share = $100.
2. 17 shares = 17 times $100 = $1,700.
3. $\frac{1}{4}\%$ of $1,700 = $4.25.

III. ∴ His brokerage was $4.25.

I. A broker received $9.50, or brokerage of $\frac{1}{4}\%$, for buying Union Pacific stock; how many shares did he purchase?

II. 1. 100% = value of shares bought.
2. $\frac{1}{4}\%$ = brokerage.
3. $9.50 = brokerage.
4. $\frac{1}{4}\%$ = $9.50.
5. $\frac{4}{4}\%$ 4 times $9.50 = $38.
6. 100% = 100 times $38 = $3,800.
7. $3,800 ÷ 100 = 38 = number of shares.

III. ∴ He bought 38 shares.

R. 3rd, p. 215.

I. A broker bought 70 shares of insurance stock at $6\frac{1}{2}\%$ premium, and sold them at $\frac{3}{4}\%$ discount; how much did he lose?

II. 1. 1 share = \$100.
2. 70 shares = 70 times \$100 = \$7,000.
3. $106\frac{1}{2}\%$ of \$7,000 = \$7,455 = cost.
4. $99\frac{1}{4}\%$ of \$7,000 = \$6,947.50 = selling price.
5. \$7,455 — \$6,947.50 = \$507.50 = loss.

III. ∴ He lost \$507.50.

I. How many shares of bank stock at 4% premium can be bought for \$8,320?

II. 1. 100% = amount of stock I can buy.
2. 4% = premium.
3. 104% = price per cent. of what I buy.
4. \$8,320 = price of what I buy.
5. 104% = \$8,320.
6. 1% = $\frac{1}{104}$ of \$8,320 = \$80.
7. 100% = 100 times \$80 = \$8,000.
8. \$8,000 ÷ \$100 = 80 = number of shares.

III. ∴ He can buy \$8,000 worth, or 80 shares.

I. How much railroad stock at $12\frac{1}{2}$ per cent. discount can I buy for \$8,320?

II. 1. 100% = amount that can be bought.
2. $12\frac{1}{2}\%$ = discount.
3. $87\frac{1}{2}\%$ = price per cent. of what I buy.
4. \$8,750 = price of what I buy.
5. $87\frac{1}{2}\%$ = \$8,750.
6. 1% = $\frac{1}{87\frac{1}{2}}$ of \$8,750 = \$100.
7. 100% = 100 times \$100 = \$10,000.

III. ∴ I can buy \$10,000 worth of stocks.

Wh. p. 157.

I. When N. Y. Central is quoted at $95\frac{3}{8}$, how much stock can I buy for \$6,894, brokerage $\frac{1}{8}$ per cent.?

II.
1. $100\% =$ amount I can buy.
2. $95\frac{5}{8}\% =$ price of stock.
3. $\frac{1}{8}\% =$ brokerage.
4. $95\frac{5}{8}\% + \frac{1}{8}\% = 95\frac{3}{4}\% =$ cost of what I buy.
5. $\$6,894 =$ cost of what I buy.
6. $95\frac{3}{4}\% = \$6,894$.
7. $1\% = \frac{1}{95\frac{3}{4}}$ of $\$6,894 = \$72$.
8. $100\% = 100$ times $\$72 = \$7,200$.

III. ∴ I can buy $7,200 worth.

Wh. p. 157.

I. A broker bought stock at 4 per cent. discount, and, selling the same at 5 per cent. premium, gained $450; how many shares did he purchase?

II.
1. $100\% =$ amount bought.
2. $4\% =$ discount on purchase.
3. $5\% =$ premium on sale.
4. $9\% =$ gain.
5. $\$450 =$ gain.
6. $9\% = \$450$.
7. $1\% = \frac{1}{9}$ of $\$450 = \$50$.
8. $100\% = 100$ times $\$50 = \$5,000$.
9. $\$5,000 \div \$100 = 50 =$ number of shares.

III. ∴ He bought 50 shares.

Wh. p. 153.

I. A man bought Michigan Central at 120, and sold at 124; what per cent. of the investment did he gain?

II.
1. $124\% =$ selling price.
2. $120\% =$ cost, or investment.
3. $124\% - 120\% = 4\% =$ gain.
4. $120\% = 100\%$ of itself.
5. $1\% = \frac{1}{120}$ of $100\% = \frac{100}{120}\% = \frac{5}{6}\%$.
6. $4\% = 4$ times $\frac{5}{6}\% = \frac{20}{6}\% = 3\frac{1}{3}\% =$ gain.

III. ∴ He gained $3\frac{1}{3}$ per cent. on the investment.

Wh. p. 158.

I. If I receive an annual dividend of 6 per cent. on Michigan Central stock which cost me 37½, what per cent. of income do I receive on investment?

II. 1. 37½% = cost or investment.
2. 6% = gain.
3. 37½% = 100% of itself.
4. 1% = $\frac{1}{37\frac{1}{2}}$ of 100% = 2.6⅔%.
5. 6% = 6 times 2.6⅔% = 16%.

III. ∴ He gained 16 per cent.

R. H., p. 218.

I. How many shares of stock, bought at 95¼ and sold at 105, brokerage ¼ per cent. on each transaction, will yield a profit of $925?

II. 1. 100% = amount that can be bought.
2. 95¼% = cost; ¼% = brokerage.
3. 95¼% + ¼% = 95½% = entire cost.
4. 105% — ¼% = 104¾% = selling price—disc't.
5. 104¾% — 95½% = 9¼% = gain.
6. $925 = gain.
7. 9¼% = $925.
8. 1% = $\frac{1}{9\frac{1}{4}}$ of $925 = $100.
9. 100% = 100 times $100 = $10,000.
10. $10,000 ÷ 100 = $100 = number of shares.

III. ∴ 100 = number of shares.

R. 3rd, p. 218.

BONDS.

When U. S. 6's are quoted at 109½, what will three $500 bonds cost?

1. $1 bonds = $1.095.
2. $500 bond = 500 times $1.095 = $547.50.

I. When gold is at 115, and U. S. 5's at \$1.05, what is a \$500 bond worth in gold?

II.
1. \$1 bond $=$ \$1.05.
2. \$500 bond $=$ 500 times \$1.05 $=$ \$525.
3. 115 cents currency $=$ 100 cents in gold.
4. 1 cent c'y $= \frac{1}{115}$ of 100 cts. gold $= \frac{20}{23}$ cts. gold.
5. 52500 cts. $=$ 52500 times $\frac{20}{23}$ cts. gold $=$ \$456.52.

III. ∴ \$456.52 $=$ value required.

REMARKS: The 1st and 2nd equations reduce the \$500 bond to currency. The 3rd, 4th and 5th equations reduce the currency to gold.

I. A college invested its endowment fund in U. S. 6's at 106; what rate of interest in currency will it receive when gold is at 120?

II.
1. 100% gold $=$ 120% currency.
2. 1% gold $= \frac{1}{100}$ of 120% c'y $= \frac{6}{5}\%$ currency.
3. 6% gold $=$ 6 times $\frac{6}{5}\%$ c'y $= 7\frac{1}{5}\%$ currency.
4. 106% currency $=$ cost of bonds.
5. $7\frac{1}{5}\% =$ gain.
6. 106% $=$ 100% of itself.
7. $1\% = \frac{1}{106}$ of $100\% = \frac{50}{53}\%$.
8. $7\frac{1}{5}\% = 7\frac{1}{5}$ times $\frac{50}{53}\% = 6\frac{42}{53}\%$.

III. ∴ The gain was $6\frac{42}{53}\%$ on the investment.

Wh. p. 205.

REMARKS: The income is 6% of the bonds **in gold**; by the first three equations we find that this 6% in gold is $7\frac{1}{5}\%$ in currency. 160% in currency is the amount paid out, and since $7\frac{1}{5}$ is the income, the gain per cent. on the investment will be whatever per cent. $7\frac{1}{5}\%$ is of 106%, which is $6\frac{4}{2}\frac{2}{3}\%$. Another way to put this, is to represent the cost as 106 cents in currency, and 6 cents in gold, or $7\frac{1}{5}$ cents currency, as the gain. Then find what per cent. $7\frac{1}{5}$ is of 106 cents.

I. What must be paid for 6% bonds to realize an income of 8%?

II.
1. 100%=what I must pay, or the investment.
2. 6% of bonds = income.
3. 8% of my investment = income.
4. 8% investment = 6% of the bonds.
5. 1% inves't=$\frac{1}{8}$ of 6% of bonds=$\frac{3}{4}$% of bonds.
6. 100% of inv.=100 tms $\frac{3}{4}$+of bds=75% of bs.

III. ∴ To make 8% I must pay 75%.

R. 3rd, p. 218.

REMARK: It must be kept in mind that I must buy the bonds so cheap that the 6% that they yield me will be 8% of what I pay for them.

I. When gold is at 115, what is the semi-annual interest in currency on $9,500 in 5's?

II.
1. $9,500 = amount of bonds.
2. 5% of $9,500=$475=annual int'est in gold.
3. $\frac{1}{2}$ of $475=$237.50=semi-annual interest.
4. $1 gold = $1.15 currency.
5. $237.50 gold=237.50 times $1.15=$273.125.

III. ∴ $273.125 = required sum.

Wh p. 205.

I. What can I afford to pay for bonds yielding an annual income of 7%, to invest my money so as to realize 6%?

II.
1. 100% = what I can pay.
2. 7% of the bonds = income.
3. 6% of my investment = income.
4. 6% investment = 7% of bonds.
5. 1% investment=$\frac{1}{6}$ of 7% bonds=$\frac{7}{6}$% bonds.
6. 100% inv't=100 times $\frac{7}{6}$% bonds=116$\frac{2}{3}$%.

III. ∴ I can afford to pay 116$\frac{2}{3}$.

COMMON FRACTIONS AND PERCENTAGE COMBINED.

1. Definitions:—

1. Common Fractions embrace all those operations in which the computation is based on the scale of 1.

2. Percentage embraces those operations in which the computation is based on the scale of 100.

It is the object of this article, to show the close relation existing between Common Fractions and Percentage.

2. Illustrations:—

I. A and B together have $1,750; A's is $\frac{3}{4}$ of B's; how much has each? Solve 1st on scale of 1, 2nd on scale of 100.

1. Solution on scale of 1.

II.
1. $\frac{4}{4}$ = B's.
2. $\frac{3}{4}$ = A's.
3. $\frac{7}{4}$ = both.
4. $1,750 = both.
5. $\frac{7}{4}$ = $1,750.
6. $\frac{1}{4} = \frac{1}{7}$ of $1,750 = $250.
7. $\frac{4}{4}$ = $1,000 = B's.
8. $\frac{3}{4}$ = $750 = A's.

III. ∴ $750 = A's.
$1,000 = B's

2. Solution on scale of 100.

II.
1. 100% = B's.
2. 75% = A's.
3. 175% = both.
4. $1,750 = both.
5. 175% = $1,750.
6. 1% = $\frac{1}{175}$ of $1,750 = $10.
7. 100% = $1,000 = B's.
8. 75% = $750 = A's.

III. ∴ $750 = A's.
$1,000 = B's.

REMARKS: (1) Observe that these solutions differ only in the scale of computation. (2) The scale determines the base—that is if the scale is one, the unit is the base, and if the scale is 100, then 100% is the base.

I. A pupil solved 8 out of 20 examination questions; what is his grade? First on scale of 100; second on scale of 1; third on scale of 10; fourth on scale of 5.

II.

FIRST—SCALE ONE HUNDRED.

1. 20 questions = 100%.
2. 1 question = $\frac{1}{20}$ of 100% = 5%.
3. 8 questions = 8 times 5% = 40% = grade.

SECOND—SCALE ONE.

1. 20 questions = 1.
2. 1 question = $\frac{1}{20}$.
3. 8 questions = $\frac{8}{20} = \frac{2}{5}$ = grade.

THIRD—SCALE TEN.

1. 20 questions = 10.
2. 1 question = $\frac{1}{20}$ of 10 = $\frac{1}{2}$.
3. 8 questions = 4 = grade.

FOURTH—SCALE FIVE.

1. 20 questions = 5.
2. 1 question = $\frac{1}{20}$ of 5 = $\frac{1}{4}$.
3. 8 questions = 8 times $\frac{1}{4}$ = 2 = grade.

III. ∴
40% = Grade on scale of 100.
$\frac{2}{5}$ = Grade on scale of 1.
4 = Grade on scale of 10.
2 = Grade on scale of 5.

REMARK: (1) Scale of 100: 20 questions answered would give the pupil a grade of 100%; hence the 1st equation 20 questions = 100%. (2) Scale of 1: In this, the pupil is entitled to a grade of 1 if he solves 20 questions; hence the 1st equation 20 questions = 1. (3) Scale of 10 and scale of 5, the same principles are involved.

SOLVE FOLLOWING.

1. The difference of two numbers is 600, and the less is 20% more than 60% of the greater; find the numbers.

2. The sum of two numbers is 800, and the greater is 20% less than 150% of the less; find the numbers

SOLVE FOLLOWING EXAMPLES.

1. A and B together have $900; A's is $\frac{4}{5}$ of B's; how much have each? Solve on scales 100, 10, 5 and 1.

2. $\frac{9}{10}$ of my money is greater than $\frac{3}{5}$ of it by $600; how much have I? Solve on scales 100, 10 and 1.

3. Sold a house for $300 on which I gained $\frac{1}{5}$ of the cost; find the cost. Solve on scales 100, 5 and 1.

4. Sold a horse for $200 which was $\frac{4}{5}$ more than the cost; find cost. Solve on scales 5 and 100.

5. Sold a horse for $100 which was $\frac{3}{4}$ more than $\frac{2}{3}$ the cost; did I gain or lose and how much? Solve on scales 1 and 100.

6. A man owns 40% of a factory and sells 10% less than 90% of his share for $500; at this rate, what is the value of the factory? Solve on scales 1, 10 and 100.

SOLVE FOLLOWING ON SCALE 100.

1. I sell goods at a gain of 10%; if they had cost $300 less, I would have gained 20%; what did the goods cost?

2. What is made on a book bought 10% below list price and sold at 45% above list price?

3. My gain is 20%; had the cost been $400 more, my loss would have been 10%; find cost.

4. I invest and sell at a loss of 15%; I invest the proceeds again and sell at a gain of 20%; how much do I gain in the two transactions?

5. How much must I invest in U. S. 5-20's at 108, so that my annual income in currency shall be $1,200 a year, if gold is worth 120?

6. Bought U. S. 6's at 120, and twice as many U. S. 5's at 115. My annual income on all was $3,000 a year in currency; if gold is worth 110, what amount did I pay for both kinds of bonds?

INTEREST.

How to Calculate Interest by 36% Method.

RULE: Take 3 times as many cents as months, plus as many mills as days; multiply this by the number of dollars in the principal. This gives gives the interest at 36%.

REMARK: If you want the interest at 9%, find interest at 36% and divide by 4; if at 12%, divide by 3; if at 6% divide by 6; if at 8% take 2-9 of interest at 36%. The advantage in this rule is the fact that the per cent can never bring you a fraction to work with.

In the 6% method we always have a bad fraction in any number of days not divisible by 6, and in the 12% method always a fraction to work with in every number of days not divisible by 3. In the 36% method no fractions, because we take exactly as many mills as days.

How to Make the Rule.

36% of $1 for 1 year is 36c.; that is, a dollar brings 36c. a year, or 3c. a month, or one mill a day. Hence take 3c. for each month and one mill for each day. If the time is years, months and days, take 36c. for each year, 3c. for each month and one mill for each day. This gives the interest on $1. On $500, 500 times this sum.

Example.

Find interest on $500 for 1 year, 11 months and 23 days at 9%.

$ .36 for 1 year.
.33 for 11 months.
.023 for 23 days.

$ .713 for whole time.
500

4) $356.500 interest at 36%.

$ 89.125 interest at 9%. Ans.

Under the subject of interest we discuss but **four cases.** Authors usually give a special rule for each of these cases, and on account of their being so much alike pupils are apt to get confused when studying the four cases together.

It is the aim here to present these cases together in such a way as to enable the student to get the processes arranged in the mind so as not to be liable to forget.

But **one** single suggestion need be stated in the way of a rule, and that is as follows: **Assume a unit of what is required, and then find the interest on the given or assumed principal, for the given or assumed time at the given or assumed rate, and compare that interest with the given interest.**

Below we give a diagram of the four cases, and it will be observed that in each case the **interest** is always given, except in the 4th case, where the **amount** is given. In that case the amount will be found and compared with the given amount.

$$\text{CASE I.—Given}\left\{\begin{array}{l}\text{Principal,}\\ \text{Rate,}\\ \text{Interest,}\end{array}\right\}\text{required Time.}$$

Assume 1 year for Time.

$$\text{CASE II.—Given}\left\{\begin{array}{l}\text{Principal,}\\ \text{Time,}\\ \text{Interest,}\end{array}\right\}\text{required Rate per cent.}$$

Assume 1% for Rate.

$$\text{CASE III.—Given}\left\{\begin{array}{l}\text{Time,}\\ \text{Per Cent.,}\\ \text{Interest,}\end{array}\right\}\text{required Principal.}$$

Assume $1 for Principal.

$$\text{CASE IV.—Given}\left\{\begin{array}{l}\text{Time,}\\ \text{Per Cent.,}\\ \text{Amount,}\end{array}\right\}\text{required Principal.}$$

Assume $1 for Principal.

EXAMPLES.—CASE I.

$$\text{I.}\quad \text{Given} \left\{ \begin{array}{l} \text{Principal \$30,} \\ \text{Rate 6\%,} \\ \text{Amount \$18.} \end{array} \right\} \text{required time.}$$

$$\text{II.} \left\{ \begin{array}{ll} 1. & \text{Assume 1 year for time.} \\ 2. & 6\% \text{ of \$30 for 1 year} = \$1.80. \\ 3. & \$18 \div \$1.80 = 10 = \text{number of years.} \end{array} \right.$$

III. $\therefore$ 10 years $=$ required time.

EXPLANATION: We assume 1 year and find, that for that time the interest would be \$1.80. If the given principal at the given rate for 1 year will give \$1.80 interest, then to yield \$18 would require as many years as \$1.80 are contained times in \$18.

CASE II.

$$\text{I.}\quad \text{Given} \left\{ \begin{array}{l} \text{Principal \$60,} \\ \text{Time 3 years,} \\ \text{Interest \$3.} \end{array} \right\} \text{required rate per cent.}$$

$$\text{II.} \left\{ \begin{array}{ll} 1. & \text{Assume } 1\%. \\ 2. & 1\% \text{ of \$60 for 3 years} = \$1.80. \\ 3. & \$3 \div \$1.80 = 1\tfrac{2}{3} = \text{the number of per cent.} \end{array} \right.$$

III. $\therefore$ $1\frac{2}{3}\%$ $=$ required rate.

EXPLANATION: 1% of the given principal for the given time gives \$1.80, but to yield \$3 would require as many per cent. as \$1.80 are contained times in \$3.

CASE III.

$$\text{I.}\quad \text{Given} \left\{ \begin{array}{l} \text{Time 3 years,} \\ \text{Rate 4\%,} \\ \text{Interest \$4.} \end{array} \right\} \text{required principal.}$$

$$\text{II.} \left\{ \begin{array}{ll} 1. & \text{Assume \$1 for principal.} \\ 2. & 4\% \text{ of \$1 for 3 years} = \$.12. \\ 3. & \$4 \div \$.12 = 33\tfrac{1}{3} = \text{number of dollars.} \end{array} \right.$$

III. $\therefore$ $\$33\frac{1}{3}$ $=$ required principal.

EXPLANATION: \$1 principal for the given time at the given rate yields \$.12 interest. Hence to yield \$4 interest will require as many dollars as \$.12 are contained times in \$4.

CASE IV.

$$\text{I. Given}\left\{\begin{array}{l}\text{Time 5 years,}\\ \text{Rate } 6\%.\\ \text{Amount } \$260\end{array}\right\}\text{required principal.}$$

$$\text{II.}\left\{\begin{array}{ll}1. & \text{Assume } \$1 \text{ for principal.}\\ 2. & \text{Amount of } \$1 \text{ for 5 years at } 6\% = \$1.30.\\ 3. & \$260 \div \$1.30 = 200 = \text{number of dollars.}\end{array}\right.$$

$$\text{III.} \therefore \$200 = \text{required principal.}$$

REMARK: These four solutions are grouped under **one plan**, and but one thing need be remembered, and that is to assume a **unit** of what is required for a starting point.

SPECIAL EXAMPLES IN PERCENTAGE.

The following problems are arranged and solved mainly for the purpose of giving the learner a drill that will fasten the idea of **percentage** upon his mind. Percentage means so many **one hundredths.**

$$1. \left\{\begin{array}{l}\frac{2}{3}\% = \text{what?}\\ \frac{2}{3}\% = \frac{2}{300} = \frac{1}{150}.\end{array}\right.$$

$$2. \left\{\begin{array}{l}\frac{2\%}{3} = \text{what?}\\ \frac{2\%}{3} = \frac{\frac{2}{100}}{3} = \frac{2}{100} \times \frac{1}{3} = \frac{1}{150}.\end{array}\right.$$

$$3. \left\{\begin{array}{l}\frac{2}{3\%} = \text{what?}\\ \frac{2}{3\%} = \frac{2}{\frac{3}{100}} = 2 \times \frac{100}{3} = \frac{200}{3} = 66\frac{2}{3}.\end{array}\right.$$

$$4. \left\{\begin{array}{l}.5\% = \text{what?}\\ .5\% = \frac{5}{10}\% = \frac{5}{1000} = \frac{1}{200}.\end{array}\right.$$

$$5. \left\{\begin{array}{l}\frac{.4}{.5}\% = \text{what?}\\ \frac{.4}{.5}\% = \frac{\frac{4}{10}}{\frac{5}{10}}\% = \frac{4}{10} \times \frac{10}{5}\% = \frac{4}{5}\% = \frac{4}{500} = \frac{1}{125}.\end{array}\right.$$

$$6. \left\{\begin{array}{l}.03\frac{1}{2}\% = \text{what?}\\ .03\frac{1}{2}\% = \frac{\frac{7}{2}}{100}\% = \frac{7}{2} \times \frac{1}{100}\% = \frac{7}{200}\% = \frac{7}{20000}.\end{array}\right.$$

BANK DISCOUNT.

1. DEFINITION: Bank discount is the simple interest on the amount due on note at settlement, from the day of discount till day of settlement.

REMARKS: (1) Bank discount is often defined as simple interest on face of note, which is not true if the note is bearing interest.

(2). In studying this subject keep in mind that bank discount is simple interest, and that to get simple interest we must have a principal, time and rate.

To illustrate take the following example:

A note of $3,000 is dated July 10, 1886, and is to run one year at 6%. It is sold at bank May 1, 1887, at a discount of 9%. Find proceeds.

Now as bank discount is simple interest on something, we must know what to calculate that interest on; that sum will be the principal the banker gets interest on. In this example it will not be the principal or face of the note, but will be the amount of the note when due.

(2). We must have time in getting simple interest, and the time in this example will be the exact number of days from the day he bought the note till it was due.

(3). In getting interest we must have rate, and the rate is 9%. The order of solution then is: (1). The banker's principal. (2). His time. (3). His rate. Then there is no trouble in securing the discount.

SOLUTION OF THE EXAMPLE :—

(1). Find amount due at settlement.

Amount of $3,000 for 1 year and 3 days at 6% is $3,181.50.

This is the banker's PRINCIPAL.

(2). Find time of discount. The note will be due July 13, 1887. It is sold May 1, and the number of days from May 1 to July 13 is 73 days.

This is the banker's TIME.

(3). His rate is 9%. The example is now one of simple interest and is this: Find interest on $3,181.50 for 73 days at 9%. The interest or bank discount is $58.06, and $3,181.50 — $58.06 = $3,123.44 = proceeds.

A note of $6,000 is dated May 20, 1884, due in two years, and bears interest at 9%; it was sold at bank Feb. 5, 1886, at a discount of 12%; what were the proceeds?

(1). Find the banker's Principal.
(2). Find the banker's TIME.
(3). His rate is 12%.

Then find simple interest and subtract from his principal.

(1). The amount of $6,000 for 2 years and 3 days at 9% = $7,084.50.

This is banker's principal.

(2). Note was due May 23, 1886, and sold Feb. 5, 1886, from Feb. 5 to May 23 is 107 days.

This is the banker's TIME.

(3). His rate was 12%.

Interest on $7,084.50 for 107 days at 12% is $252.68.
$7,084.50 — $252.68 = $6,831.82.

REMARK: If the note does not bear interest its face will be the banker's principal, for it will be the amount due at settlement.

BANKERS' EXCHANGE.

I owe John Smith, of Hamilton, O., $500. I have the money and want to send it to him. I may send it in a letter; that is dangerous. I could carry it to him; that is inconvenient and risky too. I could send by registered letter, or postoffice order. Another, and the better way, is by a bank draft. I go to my bankers, Kinney & Co., of Angola, Ind., and buy such a draft, fold it in a letter and mail it to John Smith. In a few days I get a receipt from him in full, and so far as I am concerned the obligation is discharged.

NATURE OF THIS DRAFT.

This draft is only a piece of paper, but because of what is written upon it, it has much power. The draft is an ORDER TO PAY. Kinney & Co. order, say the 10th National Bank of New York, to pay to myself, or John Smith, $500.

What authority have Kinney & Co. to ORDER the 10th National Bank of New York to pay me $500? Can they order any bank to pay? They CAN, BUT their order will not be RESPECTED, only upon the condition that they have money deposited in the bank drawn on. As an individual I can draw on any bank, but no bank will honor my order unless I have money there on deposit. Banks all over the land have money deposited in some great MONEY center like New York, or San Francisco, and it is this money they draw out by issuing drafts. This money is usually sent by express.

Let us follow this draft which I sent to John Smith, of Hamilton, O. When Smith receives this draft, let us say he takes it to the bank with which he deals, and passes it in for the money.

The steps in business are as follows:

(1). The banker at Hamilton must be certain that the person presenting the draft is really John Smith. If Smith is not known by the cashier, and the person presenting the draft claims to be that person, then the cashier requires him to identify himself, by bringing some responsible person who will say that he is the veritable John Smith. He may have signed Smith's name upon the draft, yet a forger could do that, and he must be identified. When the party is identified and the draft indorsed (signed by Smith) then the banker takes in the draft, and gives Smith or bearer the money.

Smith now drops out; the Hamilton banker now is out of $500 money and has the draft.

(2). What will this banker do to get his money? This draft, along with others he may have recently taken in he places in a letter and mails them, we will say to the 1st National Bank of New York, with which he does business (where he has money deposited), and at the same time charges this bank with $500.

The Hamilton banker now drops out, and the draft is now in the 1st National Bank of New York, and that bank has credited the Hamilton bank. The 1st National Bank does not turn this draft over to the 10th National, but turns it over the Clearing House. This House will turn it over to the 10th National the next day, and inside of six months the 10th National will return it to Kinney & Co., of Angola, Indiana.

TIME PROBLEMS.

These problems are taken from the relations of the rates of the movement of the **hour and minute hands of a clock.**

There are three sets of problems that we discuss in the three following cases: First, those in which the **distance to be gained** by the minute hand is given. Second, those in which the **sum of the distance run by both hands** is given. Third, those in which we know the distance between minute and hour hand when they start, (which is known in the above), and are required to find how far the minute hand must move to be a given part of the distance from 12 to the hour hand.

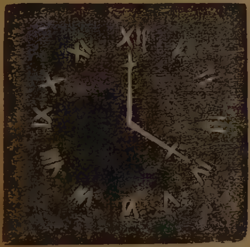
GENERAL REMARKS: 1. The minute hand always **starts at** 12; the starting point of the hour hand may be determined by noting what is the time of day required; for example, if it is to be between 3 and 4, the hour hand must start at 3; if 7 and 8, it will start at 7, and so on.

2. In solving any problem under any of the following cases, first locate the hands as stated in the above remark, and then ask yourself how far the **minute hand must move** to meet the **conditions** of the problem, **if the hour hand should remain stationary.**

3. Keep in mind that the minute hand travels 12 times as fast as the hour hand.

CASE I.

At what time between 4 and 5 will the hour and minute hands of a watch be together?



REMARK: Locate the minute hand at 12 and the hour hand at 4. The minute hand is 20 minutes behind and will have to gain 20 minutes before it overtakes the hour hand.

SOLUTION.

I. Statement of the question.

II. 1. Let $\frac{2}{2}$ = distance the hour hand moves.
2. $\frac{24}{2}$ = distance the minute hand moves.
3. $\frac{24}{2} - \frac{2}{2} = \frac{22}{2}$ = dist. gained by minute hand.
4. 20 minutes = dist. gained by minute hand
5. $\frac{22}{2}$ = 20 minutes.
6. $\frac{1}{2} = \frac{1}{22}$ of 20 minutes = $\frac{10}{11}$ minutes.
7. $\frac{24}{2}$ = 24 times $\frac{10}{11}$ minutes = $21\frac{9}{11}$ minutes.

III. ∴ $21\frac{9}{11}$ minutes past 4 = time required.

At what time between 8 and 9 will the hour and minute hand of a clock be opposite each other?

REMARK: Locate the minute hand at 12 and the hour hand at 8. When located in this way the minute hand lacks just 10 minutes of being opposite the hour hand. Hence it must gain 10 minutes.

SOLUTION.

I. Statement of the question.

II. 1. Let $\frac{2}{2}$ = distance moved by hour hand.
2. $\frac{24}{2}$ = distance moved by minute hand.
3. $\frac{24}{2} - \frac{2}{2} = \frac{22}{2}$ = dist. gained by minute hand.
4. 10 minutes = dist. gained by minute hand.
5. $\frac{22}{2}$ = 10 minutes.
6. $\frac{1}{2} = \frac{1}{22}$ of 10 minutes $\frac{5}{11}$ minutes.
7. $\frac{24}{2}$ = 24 times $\frac{5}{11}$ min. = $\frac{120}{11}$ min. = $10\frac{10}{11}$ min.

III. ∴ $10\frac{10}{11}$ minutes past 8 = time required.

The problems in this case may be solved by the following

FORMULA:

Distance moved by minute hand = $\frac{12}{11}$ of the distance to be gained by minute hand.

SOLUTION OF THE FORMULA.

1. 12 = distance moved by minute hand.
2. 1 = distance moved by hour hand.
3. 11 = distance gained by minute hand.
4. 12 = $\frac{12}{11}$ of 11. Hence the distance moved by minute hand = $\frac{12}{11}$ of the distance gained by the minute hand.

CASE II.

At what time between 5 and 6 o'clock is the minute hand the same distance from 8 as the hour hand is from 3?

PRELIMINARY REMARKS: When the hour hand is at 5, the minute hand is at 12. When situated this way, the hour hand is 10 minutes from 3. Now if the hour hand remains stationary the minute hand must move 30 minutes to be within 10 minutes of 8. The hour hand does **not** remain stationary, but moves, and moves **farther from** 3; hence the **minute hand** must stop short of 30 minutes just the same distance that the hour hand moves farther **from** 3. For example, should the hour hand move one minute it would then be 11 minutes from 3, and the minute hand must stop within 11 minutes of 8, so that it would have to travel but 29 minutes instead of 30. Thus we see that 30 minutes is the **distance moved by both.**

SOLUTION.

I. Statement of the question.

II.
1. $\frac{2}{2}$ = distance moved by the hour hand.
2. $\frac{24}{2}$ = distance moved by the minute hand.
3. $\frac{26}{2}$ = distance moved by both.
4. 30 minutes = distance moved by both.
5. $\frac{26}{2}$ = 30 minutes.
6. $\frac{1}{2} = \frac{1}{26}$ of 30 minutes $= \frac{30}{26}$ minutes $= \frac{15}{13}$ min.
7. $\frac{24}{2} = 24$ times $\frac{15}{13}$ min. $= \frac{360}{13}$ min. $= 27\frac{9}{13}$ min.

III. ∴ $27\frac{9}{13}$ minutes past 5 = time.

We let $\frac{2}{2}$ = distance moved by hour hand, then $\frac{24}{2}$ = distance moved by minute hand, because the minute hand always travels 12 times as fast as the hour hand. The (7) gives $27\frac{9}{13}$ minutes as the distance moved by the minute hand, which is the required number of minutes past 5 o'clock.

At what time between 4 and 5 will the minute hand be as far from 12 on the left side of the dial plate, as the hour hand is from 12 on the right side?

NOTE: When the hour hand is at 4, it is 20 minutes from 12. If it stood still, the minute hand would have to move round 40 minutes to 8 in order to be 20 minutes from 12. But the hour hand moves, and the distance it moves from 4, the minute hand must stop short of 8. Hence the minute hand will not move over 40 minutes, but the distance moved by both will be 40 minutes.

SOLUTION.

I. Statement of the question.

II.
1. Let $\frac{2}{2}$ = distance moved by hour hand.
2. $\frac{24}{2}$ = distance moved by minute hand.
3. $\frac{24}{2} + \frac{2}{2} = \frac{26}{2}$ = distance moved by both.
4. 40 minutes = distance moved by both.
5. $\frac{26}{2}$ = 40 minutes.
6. $\frac{1}{2} = \frac{1}{26}$ of 40 minutes $= \frac{20}{13}$ minutes.
7. $\frac{24}{2}$ = 24 times $\frac{20}{13}$ min. $= \frac{480}{13}$ min. $= 36\frac{12}{13}$.

III. ∴ $36\frac{12}{13}$ minutes past 4 = time.

METHOD BY FORMULA.

The last example may be solved by taking $\frac{12}{13}$ of 40 minutes. The one immediately preceding this, solved by taking $\frac{12}{13}$ of 80 minutes. This formula might be stated as follows: **Distance moved by minute hand** $= \frac{12}{13}$ of distance it would have to move did the hour hand remain stationary; or distance moved by minute hand $= \frac{12}{13}$ of the distance moved by both hands.

SOLUTION OF THE FORMULA.

12 minutes = distance moved by minute hand.
1 minute = distance moved by hour hand.
12 + 1 minute = 13 minutes = distance moved by both.

12 is $\frac{12}{13}$ of 13, or the distance moved by the minute hand is $\frac{12}{13}$ of the distance moved by both. But, as has already been demonstrated in the preceeding examples, the distance moved by both is the same as the distance the minute hand would move if the hour hand did not move.

CASE III.

At what time between 3 and 4 will the minute hand be $\frac{1}{3}$ of the distance from 12 to the hour hand?

REMARK: When the hour hand is at 3, minute hand is at 12. Now if the hour hand would not move, the minute hand would move just ⅓ of 15 minutes, or 5 minutes, in order to be ⅓ of the distance from 12 to the hour hand; but the hour hand moves.

SOLUTION.

I. Statement of the question.

II. 1. Let $\frac{6}{6}$ = distance moved by hour hand.
2. 15 minutes + $\frac{6}{6}$ = dist. from 12 to hour hand.
3. $\frac{1}{3}$ of (15m. + $\frac{6}{6}$) = 5m. + $\frac{2}{6}$ = dis. m. hd. moves.
4. $\frac{72}{2}$ = distance minute hand moves.
5. $\frac{72}{6}$ = 5 minutes + $\frac{2}{6}$.
(5.) − $\frac{2}{6}$ = (6.) $\frac{70}{6}$ = 5 minutes.
7. $\frac{1}{6} = \frac{1}{70}$ of 5 minutes = $\frac{1}{14}$ minute.
8. $\frac{72}{6}$ = 72 times $\frac{1}{14}$ minute = $5\frac{1}{7}$ minutes.

III. ∴ $5\frac{1}{7}$ minutes past 3 = time required.

REMARK: 1. Notice that the $\frac{6}{6}$ equals the distance moved by the hour hand; and hence all 6ths that follow are so many parts of that distance.

2. The 6th equation is gotten on the principle that the same quantity can be taken from both members of an equation and affect it only in form.

FORMULA.

If the minute hand is to stop $\frac{1}{3}$ the distance from 12 to hour hand, take $\frac{12}{11\frac{2}{3}}$ of 5; if $\frac{1}{2}$, take $\frac{12}{11\frac{1}{2}}$ of 5; if $\frac{2}{3}$, take $\frac{12}{11\frac{1}{3}}$ of 5; if $\frac{3}{4}$, take $\frac{12}{11\frac{1}{4}}$ of 5; if $\frac{1}{4}$, take $\frac{12}{11\frac{3}{4}}$ of 5, etc. Remember that the 5 is $\frac{1}{3}$ of the distance between the hand when the minute hand is at 12.

RATIO AND PROPORTION.

Ratio means relation. The ratio of 2 to 4 is ½, because 2 is ½ of 4; the ratio of 3 to 9 is ⅓, because 3 is ⅓ of 9.

Two ratios that are equal make a proportion. Thus:

$$\left.\begin{array}{l} 2:4=\frac{1}{2} \\ 3:6=\frac{1}{2} \end{array}\right\} \begin{array}{l} \text{Hence } 2:4=3:6 \text{ or} \\ \qquad 2:4::3:6 \end{array}$$

PRINCIPLES: 1. The antecedent and consequent of a ratio must be of the same denomination, because quantities of different denominations are not related.

2. If in the first couplet the antecedent is greater than the consequent, then in the second couplet the antecedent must be greater than the consequent. Also, if in the first couplet the antecedent is less than the consequent, so must the antecedent of the second couplet be less than the consequent of that couplet. This is deduced from the definition of a proportion.

3. From the second principle we deduce a third, viz: that when a proportion is written according to the above principles it will read as follows:

greater : less : : greater : less, or
less : greater : : less : greater.

4. Three terms of a proportion must be given to find a fourth.
5. The unknown term may be the 1st, 2nd, 3rd, or 4th.

EXAMPLES.

If 4 pounds of butter cost 20c. what will 6 pounds cost?

EXPLANATION: 1. A relation exists between 4 pounds and 6 pounds. Also a relation exists between 20c. and the number of cents in the answer; hence 4 pounds and 6 pounds will form one couplet or ratio, and 20c. and the answer the other ratio. Let the answer be placed, say for the first term, then 20c. will be the 2nd term. because the 1st term and 2nd term make a ratio and must be of one kind. Now the first ratio of the proportion is as follows: xc. :20c. From the nature of the question, the antecedent of this ratio is greater than the consequent, hence the same must be true of the second ratio. Therefore the proportion will be xc. :20c.: :6 :4.

2. Now let the 20c. be placed in the 1st term; then the answer will occupy the place of 2nd term, 20c. :xc. In this ratio the antecedent is less than consequent. Hence the antecedent in the second ratio must be less than the consequent, and we have 20c. :xc. : :4 : 6 From this it will appear, that what is usually placed for the 3rd term may be placed for any term.

If I walk 600 miles in 10 days, how far will I walk in 12 days?

Miles are required—place 600 miles for the 4th term, then x miles will be the 3rd, and the ratio is, x ms.: 600 ms. In this ratio the antecedent is greater than the consequent, which is seen from the nature of the question. Hence in the other couplet the antecedent will be greater than the consequent, and the proportion will be 12 : 10 : : x ms.: 600 ms., or greater is to less as greater is to less. Hence 10 x ms. = 720 ms. ∴ x ms. = 720 ms. = Ans.

EXAMPLES.

We give below, a list of examples for examination, and solve them on the following pages, for a model. We are sure that Examiners will thank us for the pages devoted to this part of the work. We think that teachers are frequently inclined to charge their failures to the Examiners when the fault is largely their own. If a teacher knows how to solve an example properly, it will pay him to arrange his work in a concise and logical manner. Many teachers have ideas with regard to the solution of problems, but they lack the power to present them in an intelligent manner.

1. A and B have $340; $\frac{3}{5}$ of A's money equals $\frac{6}{7}$ of B's; how much have each?

2. A man engaging in trade lost $\frac{2}{5}$ of his money invested, after which he gained $740, and then had $3,500; how much did he lose?

3. A person being asked the time of day said that $\frac{2}{3}$ of the time past noon equaled $\frac{3}{4}$ of the time to midnight; find the time of day.

4. A man drew 40% of his bank deposits, and expended $13\frac{1}{3}$% of the money thus drawn in the purchase of a carriage worth $116; how much money had he in bank?

5. How must I mark goods that cost $4 a yard, so that I may deduct 20% from the marked price, and still make 10%?

EXAMINATION IN ARITHMETIC.

PROBLEM I.

I. Statement of the question.

II.
1. $\frac{3}{5}$ A's money $=$ $\frac{6}{7}$ B's money.
2. $\frac{1}{5}$ A's $=$ $\frac{1}{3}$ of $\frac{6}{7}$ B's $=$ $\frac{2}{7}$ B's.
3. $\frac{5}{5}$ A's $=$ 5 times $\frac{2}{7}$ B's $=$ $\frac{10}{7}$ B's.
4. $\frac{7}{7}$ $=$ B's money.
5. $\frac{10}{7}$ $=$ A's money.
6. $\frac{7}{7}+\frac{10}{7}=\frac{17}{7}$ $=$ amount both have.
7. \$340 $=$ amount both have.
8. $\frac{17}{7}$ $=$ \$340.
9. $\frac{1}{7}=\frac{1}{17}$ of \$340 $=$ \$20.
10. $\frac{7}{7}$ $=$ 7 times \$20 $=$ \$140.
11. $\frac{10}{7}$ $=$ 10 times \$20 $=$ \$200.

III. $\therefore$ A's money $=$ \$200.
B's money $=$ \$140.

PROBLEM II.

I. Statement of the question.

II.
1. $\frac{5}{5}$ $=$ his money at the start.
2. $\frac{2}{5}$ $=$ part lost.
3. $\frac{5}{5}-\frac{2}{5}=\frac{8}{5}$ $=$ part left.
4. \$740 $=$ amount gained afterward.
5. $\frac{3}{5}$ + \$740 $=$ am't he had after gaining \$740.
6. \$3,500 $=$ amount he had after gaining \$740.
7. $\frac{8}{5}$ + \$740 $=$ \$3,500.

(7.) — \$740 $=$ (8.) $\frac{3}{5}$ $=$ \$2,760.

9. $\frac{1}{5}=\frac{1}{3}$ of \$2,760 $=$ \$920.
10. $\frac{5}{5}$ $=$ 5 times \$920 $=$ \$4,600 $=$ his money.
11. $\frac{2}{5}$ of \$4,600 $=$ \$1,840 $=$ amount lost.

III. $\therefore$ He lost \$1,840.

PROBLEM III.

I. Statement of the question.

II.
1. $\frac{2}{3}$ time past noon $=\frac{3}{4}$ time to midnight.
2. $\frac{1}{3}$ t. past n. $=\frac{1}{2}$ of $\frac{3}{4}$ t. to m. $=\frac{3}{8}$ t. to m.
3. $\frac{3}{3}$ t. past n. $=3$ times $\frac{3}{8}$ t. to m. $=\frac{9}{8}$ t. to m.
4. $\frac{8}{8}=$ time to midnight.
5. $\frac{9}{8}=$ time past noon.
6. $\frac{9}{8}+\frac{8}{8}=\frac{17}{8}=$ time from noon to midnight.
7. 12 hours $=$ time from noon to midnight.
8. $\frac{17}{8}=12$ hours.
9. $\frac{1}{8}=\frac{1}{17}$ of 12 hours $=\frac{12}{17}$ hours.
10. $\frac{9}{8}=9$ times $\frac{12}{17}$ hours $=\frac{108}{17}$ hours $=6\frac{6}{17}$ hours.

III. ∴ The time is $6\frac{6}{17}$ o'clock p. m.

PROBLEM IV.

I. Statement of the question.

II.
1. 100% = amount of money deposited.
2. 40% = amount drawn out.
3. $13\frac{1}{3}\%$ of $40\%=5\frac{1}{3}\%=$ amount spent $=\$116$.
4. $5\frac{1}{3}\%=\$116$.
5. $1\%=\frac{1}{5\frac{1}{3}}$ of $\$116=\$21.75$.
6. 100% = 100 times \$21.75 = \$2,175.

III. ∴ He had \$2,175 deposited in bank.

PROBLEM V.

I. Statement of the question.

A.
1. \$4 = cost.
2. 10% = profit.
3. 110% of \$4 = \$4.40 = selling price.

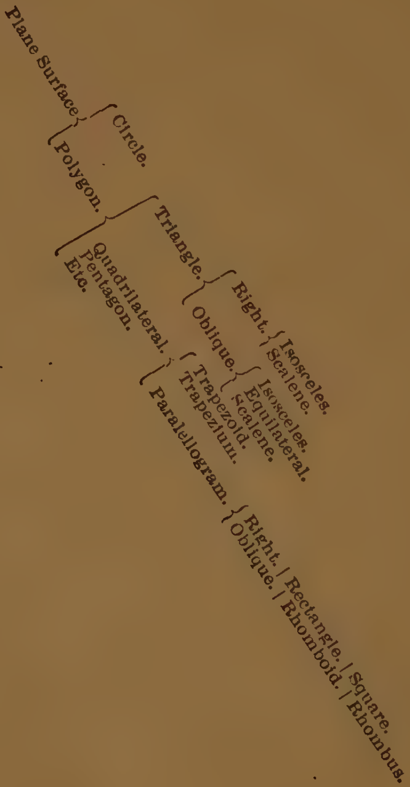
II.

B.
1. 100% = marked price.
2. 20% = deduction.
3. 100% — 20% = 80% = selling price.
4. \$4.40 = selling price.
5. 80% = \$4.40.
6. $1\%=\frac{1}{80}$ of $\$4.40=\$0.55$.
7. 100% = 100 times \$0.55 = \$5.50.

III. ∴ It must be marked at \$5.50.

MENSURATION.

DIAGRAM OF PLANE SURFACES.



The student will observe that the preceeding classification depends on definition; for instance if we should define a **square** as a **quadrilateral**, then the square should be classified immediately **under** the quadrilateral. We ought to say that a **square** is a rectangle, and for that reason it is classified under rectangle. Trapezoid is classed under quadrilateral, and should be called such in the definition. Rhombus is classed under rhomboid, and hence should be called such in definition. The old way of defining all surfaces as a plane figure is too general.

DEFINITIONS.

1. **Plane surface** is a surface such that if any two points of it be connected by a straight line, that line will lie wholly within the surface.
2. A plane polygon is a portion of plane surface bounded by any number of straight lines.
3. A circle is a portion of plane surface bounded by a curved surface, every point of which is equally distant from a point within called the center.
4. A **triangle** is a plane polygon with three sides.
5. A quadrilateral is a plane polygon with four sides.
6. A **trapezium** is a quadrilateral with no two sides parallel.
7. A trapezoid is a quadrilateral with two sides only parallel.
8. A parallelogram is a quadrilateral with its opposite sides parallel.
9. A **rectangle** is a **right** parallelogram.
10. A **rhomboid** is an **oblique** parallelogram.
11. A **square** is an **equilateral** rectangle.
12. A **rhombus** is an **equilateral** rhomboid.

REMARK: The noun that is in the predicate with the verb and its subject is the **premise** of the definition, and that noun must always express the class to which the thing defined must belong.

SOLUTION OF EXAMPLES UNDER SURFACES.

REMARK: It is not the purpose here to repeat the rules for the solution of all these surfaces but rather to present a compact **method** of reviews and to show the relation many of the surfaces hold to each other. **Let the one who would follow these solutions be sure that he knows all the rules before he begins this.**

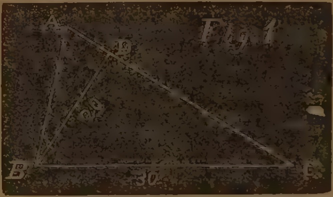
REMARK TO TEACHERS: The following development, if carried out with originality in your class, after they have the rules well in hand, will put more life into your class, and help to fix the rules and relations in their minds in a way that will surprise you if you follow it out faithfully.

THE TRIANGLE.

Rule for finding area:

Base × by ½ altitude = area.

To use this rule, or formula, the student must know the meaning of all the terms mentioned. If he is not certain that he knows what the base and altitude are, then he is uncertain about his result. But you say I know what the base is and I am never in doubt about the altitude. Take this example:



$\overline{BC}$ is 30 and B D is 20. Find area.

Solution :—

30 × 10 = 300 = area.

I have given similar examples to hundreds of teachers, and in the majority of cases, the above solution is given. It is wrong, and the pupil is misled by not knowing the meaning of the terms **base** and **altitude.**

(1.) Can $\overline{BC}$ be regarded as the base? **Yes, but if it is the base, then** $\overline{BD}$ **is not the altitude.**

(2.) Can $\overline{BD}$ be regarded the altitude? Yes, but if it is, then B C is **not the base, but** $\overline{AC}$ **is the base.**

(3.) **Any** side may be taken as the **base,** and the perpendicular from the opposite angle to this side or the side **extended,** must be the altitude. For instance if I take $\overline{AC}$ as the base, then $\overline{BD}$ is the altitude. If I take $\overline{BC}$ as the base, then a perpendicular from A to B C will be the altitude. This problem **cannot be solved,** because I have not the **base** and altitude of the triangle. If I say 30 is the base then I have no altitude. If I say 20 is the altitude then I have no base.

EXAMPLES.

(1.) B C is 40 and a perpendicular from C to A B is 30; find area. This cannot be solved.

(2.) B C is 60 and a perpendicular from A to B C is 30; find area.

Base × by ½ altitude = area.
Or 60 × 15 = 900 = answer.

(3.) Area of the triangle A B C is 900; B C is 60; find altitude.

Base × ½ altitude = area, or
60 × ½ altitude = 900, or
½ altitude = 900 ÷ 60 = 15. Hence
Altitude = 30.

(4.) Area of A B C is 1,200, and perpendicular from C to A B is 30; find A B.

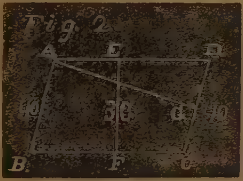
Base × ½ altitude = area, or
Base × ½ of 30, or 15 = 1,200. Hence
Base = 1,200 ÷ 15 = 80.

(5.) Area of A B C is 600 and B C is 30, find perpendicular distance from B to $\overline{AC}$. It cannot be done.

REMARK TO TEACHER: In the formula base×by ½ altitude = area are three things; teach the pupils that if any two of them are known the other can be found. Put pupils at black-board and make examples for them and let them apply this formula. You will be surprised at their success.

THE PARALELLOGRAM.

Rule for finding area. Base × altitude = area. Here too we must know the meaning of base and altitude. The base is not necessarily the side toward the ground. Look at this figure:



Here the student would be in danger of getting 1,200 for the area.

This parallellogram cannot be solved, because I have not the base and altitude. I may call 40 the base, but in that event, I have no altitude. I may call 30 the altitude, but in that event I have no base. If I call B C the base then E F is the altitude. If I call A B the base then a perpendicular between A B and D C would be the altitude as the line A G.

EXAMPLES.

(1.) B C is 60, and E F is 20; find area:
B × altitude = area, or 60 × 20 = 1,200.

(2.) A B is 40 and $\overline{AG}$ is 50; find area:
B × altitude = area, or 40 × 60 = 2,400 = ans.

(3.) A B is 40, B C is 50, D C is 40 and A D 50; find Area.

This cannot be solved. We have bases, but no altitude. It is evident that this figure may have this same boundary but have at the same time various areas. If we make it more right, that is nearer a rectangle it will have same boundary, but more area; if we make it more oblique it still may have same boundary, but a less area, and the area may grow less and less, until the opposite sides lie together and there is no surface whatever.

RECTANGLES.

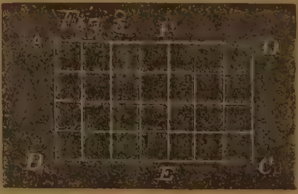
RULE: As a rectangle is a paralellogram the rule for each is the same.

EXAMPLES.

A rectangular field contains 70 acres, and its breadth is to its length as 4 to 7; find length and breadth.

SOLUTION.

Let us draw a rectangle whose breadth is to its length as 4 to 7. Here it is:



This figure is 7 parts long and 4 parts wide. If we pass from A to the right and count four parts, and letter it A B E F, this figure will be a SQUARE, and will contain $\frac{4}{7}$ of the whole area, or $\frac{4}{7}$ of 70 acres which is 40 acres.

40 acres $=$ 6,400 square rods.

$\sqrt{6{,}400} = 80$ rods $=$ A B. If $\overline{AB}$ is 80 rods, then $\overline{A\,D}$ is 170 rods.

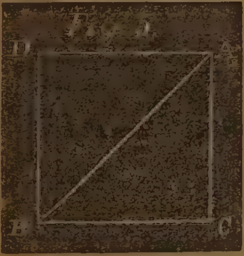
REMARK: The ratio of the breadth to length in this example was 4 to 7, and the square A B E F was $\frac{4}{7}$ of the whole field. This principle enables us to shorten such examples as this as follows: If a rectangle contains 81 square rods, and breadth is to length as 4 to 9—then if we take $\frac{4}{9}$ of $81 = 36$, this 36 will be the square, and its root 6 will be the width, and $\frac{9}{4}$ of $6 = 13\frac{1}{2}$ is the length. Again, if a rectangle whose breadth is to length as 3 to 11 has an area of 297, then the square is $\frac{3}{11}$ of 2,97 square rods $=$ 81 square rods and the square root of $81 = 9$ rods the breadth, and the length is $\frac{11}{3}$ of $9 = 33$ rods the length.

THE RIGHT TRIANGLE.

1ST. LAW.

$h^2 = p^2 + b^2$, or if p and b are equal $h^2 = h^2 = 2\,p^2$ or $2\,b^2$.

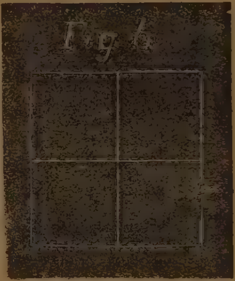
THE SQUARE.

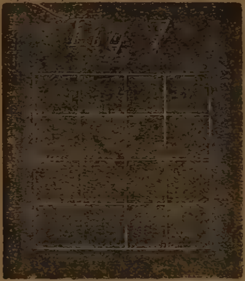


A B is the diagonal of the square, or the hypothenuse of the right triangle A B C.

In this case $\overline{AB}^2 = 2\,\overline{BC}^2$ or $2\,\overline{AC}^2$.

PRINCIPLE A.—This formulates the principle that the square of the diagonal of a square, is equal to twice the square of one side, or twice the square itself.





PRINCIPLE B.—It will be seen by Fig. 6 that the square of half a line is equal to ¼ the square of the whole line. Also from Fig. 7 that the square of ¼ a line is equal to $\frac{1}{16}$ the square of the whole line. Further, it should be kept constantly in the mind that the square of ⅔ of a line is $\frac{4}{9}$ the square of the whole line and that of ¾ of a line is equal to $\frac{9}{16}$ the square of the whole line, etc. This, for convenience, we call **Principle B.** Let us try some small examples by Principle A and Principle B.

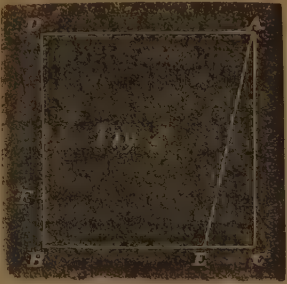
EXAMPLE.

(1.) If the square A D B C has 40 acres in it, find the line A B. Use Fig. 5.

$\overline{AB}^2 = 80$ acres, or 12,800 sq. rods, Prin. A.

$\overline{AB} = \sqrt{12,800} = 113 +$ rods.

(2.) Again, if the square, Fig. 8, contain 40 acres, find the distance from A to a point $\frac{1}{4}$ the distance from C to B as E.



SOLUTION.

The line A E belongs to a right triangle, and if we could know the squares of the other sides A C and E C we would add these together and take square root.

$\overline{AC}^2 = 40$ acres or 6,400 square rods.
$\overline{EC}^2 = \frac{1}{16}$ of $6{,}400 = 400$ square rods. Prin. B.
Hence $\overline{AE}^2 = 6{,}800$ and $AE = \sqrt{6{,}800} = 82+$.

(3.) If the square, Fig. 8, contains 6,400 square rods, find distance from E to D.

SOLUTION: If we should draw a line from E to D we should have a right triangle. Hence let us get this square of B E and square of D B.

$\overline{DB}^2 = 6{,}400$.
$\overline{BE}^2 = \frac{9}{16}$ of $6{,}400 = 3{,}600$. Prin. B.
Hence, $\overline{ED}^2 = 10{,}000$.
$\overline{ED} = \sqrt{10{,}000} = 100 =$ answer.

(4.) If the square, Fig. 8, contains 4,800 find a line from E to a point one-fourth distance above B as F.

SOLUTION: We would have another right triangle.

$\overline{BE}^2 = \frac{9}{16}$ of $4{,}800 = 2{,}700$.
$BF^2 = \frac{1}{16}$ of $4{,}800 = 300$.
Hence $\overline{EF}^2 = 3{,}000$.
$\overline{EF} = \sqrt{3{,}000} = 54+$.

(5.) The diagonal of the square, Fig. 8, is 80; find a line from F to middle point of $\overline{AE}$.

SOLUTION: Let learner draw figure on paper as now directed: Put P at the middle point of A E; also put Q at middle of D B, then draw line from P to Q; also a line from P to F. We now have a right triangle P Q F.

$\overline{QF}$ is $\frac{1}{4}$ of $\overline{DB}$ and $\overline{FQ}$ is $\frac{3}{4}$ of D B $+ \frac{1}{8}$ of D B $= \frac{7}{8}$ of D B.

Now $80^2 = 6{,}400$; $\frac{1}{2}$ of $6{,}400 = 3{,}200 = \overline{DB}^2$ by Prin. (A).

$\overline{PQ}^2 = \frac{49}{64}$ of $3,200 = 2,450$ by Prin. (B).
$Q\,F^2 = \frac{1}{16}$ of $3,200 = 200$. " " "
Hence $\overline{FP}^2 = 2,650$.
$F\,P = \sqrt{2,650} = 51+$.

(6.) The diagonal of the square, in Fig. 8, is 80, find distance from the point C to a point on E A ¼ distance from E.

SOLUTION: Place O at the point named and draw line from C to O. Then let a perpendicular fall from O to line E C, mark the point R. We shall now have the right triangle O R C.

O R is ¼ of B C.
R C is $\frac{3}{16}$ of B C.

Now $80^2 = 6,400$; ½ of $6,400 = 3,200$ $\overline{BC}^2 =$ Prin. (A).
$\overline{OR}^2 = \frac{1}{16}$ of $3,200 = 200$. Prin. B.
$\overline{RC}^2 = \frac{9}{256}$ of $3,200 = 132.5$. Prin. B.
$\overline{OC}^2 = 332.5$.
$\overline{OC} = \sqrt{332.5} = 18$.

(7.) Find in the above example the distance from O to the center of the square.

CONSTRUCTION: Let F be the center. Draw a horizontal line through the center of the square, and draw a perpendicular from O to this line and mark the place of intersection by X. We now have a right triangle O X F.

O X is ¼ B C. F X is ¼ of B C. $80^2 = 6,400$; ½ of $6,400 = 3,200$. Prin. A.
$\overline{OX}^2 = \frac{1}{16}$ of $3,200 = 200$. Prin. B.
$\overline{FX}^2 = \frac{1}{16}$ of $3,200 = \underline{200}$. Prin. B.
$\overline{FO}^2 =$ 400. Hence $F\,O = 20$. Ans.

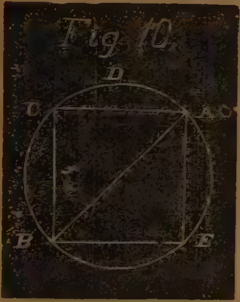
The diameter of the circle is same as the side of the square.

RULE FOR CIRCLE.

$\text{Diameter}^2 \times .7854 =$ area of circle, or what is substantially the same, $\text{diameter}^2 \times \frac{4}{5} =$ area of circle in the square.

$\text{Diameter}^2 =$ area of the square.

From this we see that A CIRCLE IS $\frac{4}{5}$ OF THE SQUARE ABOUT IT. Call this (PRIN. C).



$\overline{AB}^2 =$ twice the square by (Prin. A).

Hence $\frac{1}{2}\,\overline{AB}^2 =$ the square.

$\overline{AB}^2 \times \frac{4}{5} =$ area of circle by rule $(.7854 = \frac{4}{5})$.

Hence if the square is $\frac{1}{2}\,\overline{AB}^2$ and the circle is $\frac{4}{5}\,\overline{AB}^2$, then the square and circle are related as $\frac{1}{2}$ and $\frac{4}{5}$. Now $\frac{1}{2}$ is $\frac{5}{8}$ of $\frac{4}{5}$.

Hence A SQUARE IS $\frac{5}{8}$ OF A CIRCLE ABOUT IT. Call this (PRIN. D).

EXAMPLES.

(1.) The corner (Fig. 9) A B C contains 4 acres; find the area of the circle, also of the square.

SOLUTION: The four corners contain 16 acres, and this is the difference between the circle and the square. $\frac{5}{5} =$ the square.

By Prin. C the circle is $\frac{4}{5}$ of the square. Hence $\frac{4}{5}$ the circle.

$\frac{1}{5}$ 16 = acres.

$\frac{5}{5}$ = 80 acres = the square.

$\frac{4}{5}$ = 64 acres = the circle.

(2.) The corner A B C, Fig. 9, contains 20 square rods; find a straght line from A to C.

$\frac{5}{5}$ = square.
$\frac{4}{5}$ = circle.
$\frac{1}{5}$ = 80 square rods = the four corners.
$\frac{5}{5}$ = 400 square rods = area of square.
$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$.
$\overline{AB}^2 = \frac{1}{4}$ of 400 by Prin. B. $\overline{BC}^2 = 100$ by Prin. B.

Hence $\overline{SC}^2 = 200$.
$AC = \sqrt{200} = 14+$.

(3.) In Fig. 9 the diameter of circle is 20; find the corner A B C.

SOLUTION.

$20^2 \times .7{,}854 = 319$ = area circle.
$\frac{5}{5}$ = square.
$\frac{4}{5}$ = circle = 319.
$\frac{1}{5} = \frac{1}{4}$ of 319 = 79.7 = the four corners.
One corner = $\frac{1}{4}$ of 79.9.

(4.) In Fig. 9 the square is 40 acres; find corner A B C.

$\frac{5}{5}$ = square = 40 acres.
$\frac{1}{5}$ = 8 acres = the corners.
One corner = $\frac{1}{4}$ of 8 = 2 acres = answer.

(5.) In Fig. 9, if the circle contains 20 acres find shortest distance from B to the circumference of circle.

(6.) In Fig. 9, if a straight line from A to C is 20 rods, find area of the corner A B C.

SOLUTION COMMENCED: $\overline{AC}^2$ or $\overline{20}^2 = 400 =$ ½ area of the square by Prin. A and Prin. B. Let learner finish.

(7.) In Fig. 10 the diagonal is 40; find the area of circle.

SOLUTION: $40^2 = 1{,}600$; $1{,}600 \div 2 = 800$ = the square by **Prin. A.** By **Prin. D** the square is $\frac{5}{8}$ of the circle.

$\frac{8}{8}$ = circle.
$\frac{5}{8}$ = square = 800.
$\frac{1}{8} = \frac{1}{5}$ of $800 = 160$.
$\frac{8}{8} = 1{,}280$ = circle = answer.

(8.) In Fig. 10 the diagonal is 80; find the segment A D C.

$80^2 = 6{,}400$; $6{,}400 \div 2 = 3{,}200$ = the square by Prin. A.

$\frac{8}{8}$ = circle.
$\frac{5}{8}$ = square.
$\frac{3}{8}$ = the four segments.
$\frac{5}{8} = 3{,}200$.
$\frac{1}{8} = 640$.
$\frac{3}{8} = 1{,}920$ = four segments.
$\frac{1}{4}$ of $1{,}920 = 480$ = answer.

(9.) In Fig. 10, A B is 60; find the arc A D C.

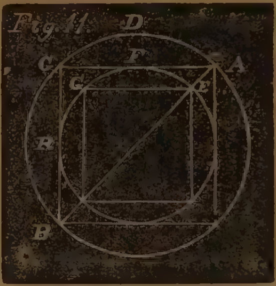
$AB^2 = 3{,}600$.
$3{,}600 \div 2 = 1{,}800$ = the square by Prin. A.
$\frac{5}{8} = 1{,}800$. Prin. D.
$\frac{1}{8} = 360$.
$\frac{8}{8} = 2{,}880$.

Having the area of the circle, its circumference may be found by the ordinary rule. The arc required is $\frac{1}{4}$ of the circumference.

(10.) In Fig. 10, draw a line from the middle of A C to the middle of A E. Given this line 40 to find the segment A D C.

$40^2 = 1{,}600$.
$1{,}600 \times 2 = 3{,}200$ = the square.

From this the segment can readily be found by **Prin. D.**



Here are two circles and two squares. We know by Principles "C" and "D" how the circles and squares are related, but the question now to be decided is what relation do the circles hold to the circles, and the squares to the squares?

REMARK: We shall use $\frac{4}{5}$ for .7854.

In Fig. 11, $\overline{AB}^2 \times \frac{4}{5} =$ area of large circle by Prin. A.

$\frac{\overline{AB}^2}{2} \times \frac{4}{5} =$ area of smaller circle.

Here is explanation for last statement: $\frac{\overline{AB}^2}{2} =$ area of larger square, but the smaller circle is $\frac{4}{5}$ of that by Prin. A.

It is evident from the above that the smaller circle is ½ of the larger. Hence

(Prin. E.) The inner circle is ½ the next outer and so on out.
The inner square is ½ the next outer and so on out.
The inner segment G F E is ½ the outer C D A and so on out.
The inner corner R C F G is ½ the outer and so on.

COROLLARY: The square of the diagonal of any square is the area of the next largest square.

(11.) In Fig. 11 the diagonal of the innermost square is 40; find a segment of the outermost circle.

SOLUTION.

$40^2 = 1,600 =$ twice the inner square (Prin. A.) or once the second square by Prin. E. But the second square is $\frac{4}{5}$ of the outermost circle, hence $\frac{4}{5} = 1,600$.

$\frac{1}{5} = 400 =$ the sum of the four segments. Hence one segment is ¼ of $400 = 100$.

Proof: 400 = the segments = $\frac{1}{5}$ of the outer circle.

$\frac{4}{5}$ = 1,600 = larger square = the square of the diagonal of inner square by corollary under Prin. E. $\sqrt{1,600}$ = 40 = the diagonal.

(12.) In Fig. 11 the diagonal of the inner square is 20; find the corner F C R G.

SOLUTION.

20^2 = 400 = the second square by corollary under Prin. E. But the inner circle is $\frac{5}{8}$ of this square by Prin. C.

Hence $\frac{8}{8}$ = the square = 400.
$\frac{1}{8}$ = 50.
$\frac{3}{8}$ = 150 = the four corners.
One corner = $\frac{1}{4}$ of 150 = 37$\frac{1}{2}$ = answer.

(13.) In Fig. 11 there are but two squares; this could be extended to an indefinite number.

EXAMPLE.

If the diagonal of the fifth square is 80, find the inner segment E F G.

SOLUTION.

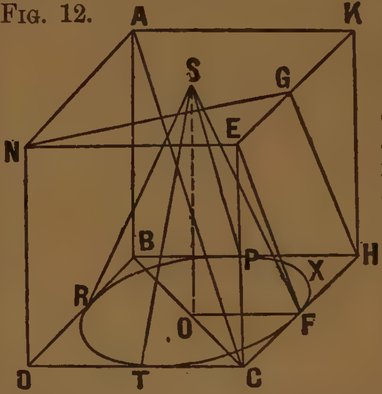
80^2 = 6,400 = twice the fifth square; 6,400 ÷ 2 = 3,200 the fifth square by Prin. E. 3,200 ÷ 2 = 1,600 = the fourth square 1,600 ÷ 2 = 800 = third square. 800 ÷ 2 = 400 = second square; 400 ÷ 2 = 200 = first square all by Prin. E. But the first square is $\frac{4}{5}$ of the first circle. $\frac{4}{5}$ = 200; $\frac{1}{5}$ = 50 = the four segments. Hence one segment = $\frac{1}{4}$ of 50 = 12$\frac{1}{2}$ answer.

REMARK: As there were 5 squares the inner would be $\frac{1}{32}$ of the outer. If there were six the inner would be $\frac{1}{64}$ of the outer and so on.

SOLIDS.

The cube has six equal square faces. Any edge of the cube squared will give one of these faces.

FIG. 12.



For example, $\overline{BD}^2$ = one face, or $\overline{DC}^2$ = one face, or $\overline{AB}^2$ = one face.

PRINCIPLE A¹: The diagonal of a cube *squared*, equals three faces of the cube, or 3 times the square of one edge.

DEMONSTRATION: B D C is a right triangle. A B C is a right triangle. Hence, $\overline{BC}^2=\overline{BD}^2+\overline{DC}^2$. Also, $\overline{AC}^2=\overline{AB}^2+\overline{BC}^2$. Now instead of $\overline{BC}^2$, place its equal $\overline{BD}^2+\overline{DC}^2$, and we have $\overline{AC}^2=\overline{AB}^2+\overline{BD}^2+\overline{DC}^2$. But $\overline{AB}^2$, $\overline{BD}^2$ and $\overline{DC}^2$ are respectively faces of the cube. Hence $\overline{AC}^2$ is equal to three faces.

REMARK: $\overline{BC}^2$ = two faces.

EXAMPLES.

(1.) If AC in Fig. 12 is 40, find the surface of the cube.

SOLUTION.

$40^2=1{,}600=3$ faces, Prin. A¹
Hence 6 faces $=3{,}200=$ answer.

(2.) If AC = 30, find BD.

SOLUTION.

$30^2=900=3$ faces, Prin. A¹
$\frac{1}{3}$ of $900=300=1$ face.
$\sqrt{300}=17.3=$ B D.

(3.) The surface of the cube is 7,200, find A C. $7,200 \div 2 = 3,600 = 3$ faces $=$ the square of AC by Prin. A^1.

$$\sqrt{3,600} = 60 = \overline{AC}.$$

(4.) $\overline{AC} = 60$; find $\overline{EF}$.

$\overline{AC}^2 = 3,600$; $3,600 \div 3 = 1,200 = \overline{EC}^2$ by Prin. A^1. $\overline{CF}^2 = \frac{1}{4}$ of $1,200 = 300$ by Prin. B under the square $\sqrt{1,200 + 300} = 38 +$.

(5.) $\overline{AC} = 60$; find area of a circle inscribed in one face of the cube.

$\overline{AC}^2 = 3,600$; $3,600 \div 3 = 1,200 =$ area of one face. Now take .7854 of this and we have the area of the circle, or to be brief and not so exact, take $\frac{4}{5}$ of 1,200.

(6.) $\overline{AC}$ is 30; find shortest distance from N to H on the surface.

The shortest path will be along the line N G H. This is easily seen by regarding a chalk box as a cube and marking it off as in this figure. Then revolve the lid on E K as a hinge until N is in a straight line with E C. Then N G H will be a straight line and the hypothenuse of a right triangle of which N E C will be altitude and C H the base. $\overline{AC}^2 = 900$; $900 \div 3 = 300 = \overline{CH}^2$. N E C is twice as long as C H; hence its square will be 1,200; $300 + 1,200 = 1,500$; $\sqrt{1,500} = 38 + =$ N G F the shortest distance.

(7.) Given A C, in Fig. 12, 30; find slant hight of cone.

SOLUTION.

We want to find the line S F. We shall get it through the right triangle S O F. If we know the square of S O and the square of O F we can get S F.

$30^2 = 900 = 3$ times $\overline{SO}^2$ by Prin. A^1.

$\frac{1}{3}$ of $900 = 300 = \overline{SO}^2$. $\overline{OF}^2$ is $\frac{1}{4}$ of $300 = 75$ by Prin. B.

$300 + 75 = 375 = \overline{SF}^2$. $\sqrt{375} = 19.4 = \overline{SF}$.

(8.) Given the irregular corner H P X F 20 square rods to find capacity of the cone.

SOLUTION.

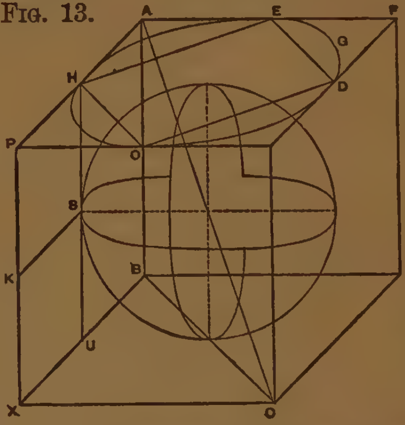
To get the capacity we must know the base and altitude. The four corners = 80 square rods, which is $\frac{1}{5}$ of the square H B D C by Prin. C under Fig. 9. If $\frac{1}{5} = 80$, $\frac{4}{5} = 320 =$ the circle by the same principle. We now have the base.

$\frac{5}{5} = 400 =$ the square H B D C by Prin. C.

$\sqrt{400} = 20 =$ altitude. Hence $\frac{320 \times 20}{3} = 2{,}133\frac{1}{3}$ by the rule for getting capacity of a cone.

Let the teacher make more examples and solve them.

Fig. 13.



Here is a sphere in a cube. Let us study these two solids together:

(1.) The diameter of the sphere is the edge of the cube.

(2.) The great circle of the sphere is the same as the circle inscribed in one of the faces of the cube.

(3.) Diameter^2 = one face of the cube, and this multiplied by .7854 will give the circle inscribed in one face. Here is the statement:

(a) Diameter$^2 \times .7854 =$ inscribed circle.

(b) Diameter$^2 \times 3.1416 =$ surface of the sphere by rule. Now this last statement is just 4 times the one marked (a).

HENCE THE SURFACE OF A SPHERE IS EQUAL TO 4 TIMES THE CIRCLE INSCRIBED WITHIN ONE FACE OF THE CUBE.

This we shall call PRIN. B^1.

NOTE: Here is another interesting relation. $\text{Diameter}^2 \times 3.1416$ = surface of sphere. $\text{Diameter}^2 \times 6$ = surface of the cube. Hence the surface of the cube is nearly twice the surface of the sphere, practically 1.9 times that of the sphere.

(1.) In Fig. 13, $\overline{BC} = 40$; find surface of inscribed sphere.

SOLUTION.

$40^2 = 1{,}600$ = two faces by Prin. A. ½ of 2,600 = 800 one face. $.7854 \times 800 = 628.32$ = area of one of the inscribed circles. But as the sphere is equal to four such circles, then $628.32 \times 4 = 2513.28$ = surface of sphere.

(2.) The surface of the inscribed sphere is 12,000; find surface of the cube.

SOLUTION.

$12{,}000 \times 1.9 = 22{,}800$ answer.
See note under Prin. B^1.

(3.) In Fig. 13, the irregular figure D F E G contains 5 acres; find number of acres in the surface of the cube.

SOLUTION.

By Prin. C under the square we have $\frac{5}{5}$ = the square F P, and $\frac{4}{5}$ = the inscribed circle $\frac{1}{5}$ = the four irregular corners = 20 acres, $\frac{5}{5} = 100$ acres = one face of cube. Hence 600 acres is surface of cube.

(4.) In Fig. 13 the irregular figure D F E G is 10 acres; find surface of inscribed sphere.

SOLUTION.

Prin. C gives $\frac{5}{5}$ = square F P. $\frac{4}{5}$ = circle $\frac{1}{5} = 40$, the four corners. $\frac{4}{5} = 160$ = the circle. But by Prin. B^1 under Fig. 13, the surface of the sphere is 4 times this circle. Hence surface of the sphere is $160 \times 4 = 640$ acres = answer.

(5.) In Fig. 13 the segment D G E is 15 acres; find surface of cube.

SOLUTION.

By Prin. D under Fig. 7 the square D E H O is $\frac{5}{8}$ of the circle about it. Hence $\frac{8}{8}$ = the circle. $\frac{5}{8}$ = the square D H $\frac{3}{8}$ = the four segments = 60 acres, $\frac{3}{8}$ = 60 acres, $\frac{1}{8}$ = 20 acres and $\frac{8}{8}$ = 160 acres = area of circle D G H O. But by Principle C under Fig. 7 this circle is $\frac{4}{5}$ of the square F P. Hence $\frac{5}{5}$ = square, and $\frac{4}{5}$ = the circle = 160 of $\frac{4}{5}$ = 160 then $\frac{1}{5}$ = $\frac{1}{4}$ of 160 = 40 and $\frac{5}{5}$ = 800 = the square F P. But the surface of the cube is 6 times 800 = 4,800 = answer.

(6.) In Fig. 13, D G E is 10 acres; find surface of inscribed sphere:

SOLUTION.

By Prin. D Fig.7, $\frac{8}{8}$ = circle and $\frac{5}{8}$ = the square D E H O and $\frac{8}{8}$ = the four segments = 40 acres. If $\frac{3}{8} = 40$, $\frac{1}{8} = \frac{40}{3}$ and $\frac{8}{8} = \frac{320}{3}$ acres = area of the circle. But the surface of the sphere is 4 times $\frac{320}{3} = \frac{1280}{3}$ = $426\frac{2}{3}$ acres, by Prin. B^1 Fig. 13.

(7.) In Fig. 13, E D = 40 rods; find capacity of inscribed sphere:

SOLUTION.

To find the capacity of sphere we should know the diameter.

ED^2 or $40^2 = 1600$ = twice the square of E F by **Prin. A.** under Fig. 2. But AF^2 is 4 times EF^2 by Prin. B under Fig. 3. Hence $AF^2 = 3200$ and $\sqrt{3200} = 56.5$ = diameter of sphere. $56.5^3 \times .5236 = 94666.88$.

(8.) In Fig. 13, E D = 40; find surface of inscribed sphere:

SOLUTION.

$\text{Diamater}^2 \times 3.1416$ = surface by rule. ED^2 or $40^2 = 1600$ = twice $EF^2 = \frac{1}{2}\ AF^2$. Hence A F = 3200. But AF^2 is the square of the diameter. Hence $3200 \times 3.1416 = 10053.12$ = answer.

PROBLEMS TO BE SOLVED.

1. If in Fig. 13, the square K S U X contains 20 square feet, what is the diagonal A C?

2. Given the circle inscribed in K S U X to be 20; find surface of inscribed sphere.

3. Given diagonal A C = 60 feet. Find diagonal of the square K S U X.

4. Given diagonal of square D E H O 60; find capacity of inscribed sphere.

5. Given capacity of inscribed sphere 10472, to find A C.

6. Given the segments D G E 10 acres; to find surface of inscribed sphere.

7. Given the irregular surface D G E F 10 acres; to find capacity of the cube.

8. If the segment D G E is 20 acres, find the surface of a sphere circumscribed about this cube.

9. If the square K X U S is 10 acres, find the surface of the sphere circumscribed about this cube.

Principle C¹ A sphere circumscribed about a cube is 5.2 times the sphere incribed within it.

Pupils not familiar with algebra are not expected to understand the reason why, but we here give a demonstration for such as may be interested.

In Fig. 13, let A C be represented by D, and A B by d. D is the diameter of the outer sphere, and d is diameter of the inner. By rule for capacity of a sphere, found in all arithmetics—

(1) $D^3 \times .5236 =$ capacity of outer sphere.
(2) $d^3 \times .5236 =$ capacity of inner sphere.

We cannot compare these in this form because one is in the terms of D and the other in terms of d. The plan is to reduce one to terms of the other as follows :

By Prin. A^1 Fig. 12, $D^2 = 3\,d^2$ from which

$$D = d\sqrt{3} = 1.7323\,d.$$

$$D^3 = 5.198395\,d^3.$$

Substitute this in equation (1) above and we have

$5.198395 \times .5236\,d^3 =$ capacity of outer sphere, or

(3) $2.721879\,d^3 =$ capacity of outer sphere.

Equa. (2) becomes (4) $5236\,d^3 =$ cap. inner sphere.

$(3) \div (4) = 5.19$. Hence the outer is 5.19 tms the inner.

EXAMPLES.

1. In Fig. 13 the area of a circle D G H O is 40 acres; find surface of circumscribed sphere.

SOLUTION.

By Prin. B^1 under Fig. 10 the inner sphere is 40 acres, and by Prin. C^1 the outer sphere is $40 \times 5.19 = 207.6$ acres = answer.

REMARKS. This principle is not valuable, only as a means of getting a wider understanding of the various relations existing between these solids. All examples solved by this law could be solved through A C. In fact, A C is the key to these solids.

THE SQUARE PYRAMID AND CYLINDER.

A pyramid or cylinder may be inscribed in the cube and examples made as with other solids. The relation is such that hundreds of very interesting examples may be made, that make a fine review of all the rules pertaining to these solids.

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